

# MCAT<sup>®</sup> EXCELERATOR<sup>™</sup>

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## Chapter 2

# MCAT Quantitative Skills

The bad news is that you can't use a calculator on the MCAT. The good news is you won't need to. Almost all questions are conceptual, and those that do require computation, don't require much. So how much math do you need? Some basic quantitative skills, an understanding of units, a bit of geometry, and a healthy helping of trigonometry. Enjoy... [0-].

## 2.1 Mathematics

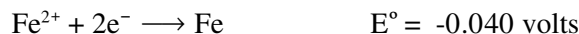
### A. Magnitude

■ The magnitude (size) of a number increases as one moves from left to right below:



#### assignment ►

Which of these two half reactions has the larger standard reduction potential?



#### solution:

The first reaction does because  $-0.040 > -1.706$  ♦

#### assignment ►

List the following compounds in order of increasing solubility (as  $K_{\text{sp}}$  increases, solubility increases).



*solution:*



■ When working in scientific notation, if the number to the left of the  $\times 10^{exp}$  gets smaller, the *exponent* gets larger.

*assignment* ►

Convert the wavelength of red light (700 nm) to meters using scientific notation (1 nm =  $10^{-9}$  m).

*solution:*

$$700 \text{ nm} = \underbrace{700}_{\text{n}} \times \underbrace{10^{-9}}_{\text{m}} \text{ m} = 7.00 \times 10^{-7} \text{ m} \quad \blacklozenge$$

■ When a unit becomes larger, the number attached to that unit becomes smaller, and visa versa.

*assignment* ►

How many cubic centimeters are there in 7.5 liters? One cc is equal to one milliliter. The prefix "milli" means one over a thousand.

*solution:*

Since the unit of measurement is getting *smaller* by 1/1000, (liter to milliliter), the number attached to the unit milliliter must get *larger* by a thousand. Thus there are  $7.5 \times 1000$ , or 7500 mL in 7.5 L.

$$\frac{7.5 \text{ L}}{1} \times \frac{1000 \text{ mL}}{1 \text{ L}} = 7500 \text{ mL} \quad \blacklozenge$$

## B. Algebra

*assignment* ►

What is the sum of  $1.5 \times 10^{-3} + 5.0 \times 10^{-4}$ ?

*solution:*

First convert the exponent of the smaller number to equal the exponent of the larger number. Then add.

$$1.5 \times 10^{-3} + 0.50 \times 10^{-3} = 2.0 \times 10^{-3} \quad \blacklozenge$$

*assignment* ►

Solve for °C in the equation:  $^{\circ}\text{F} = \frac{9}{5}^{\circ}\text{C} + 32 .$

*solution:*

First subtract 32 from both sides of the equation to yield  
 $^{\circ}\text{F} - 32 = \frac{9}{5}^{\circ}\text{C}$ .

Next divide both sides of the equation by 9/5 to give  
 $\frac{5}{9}(^{\circ}\text{F} - 32) = ^{\circ}\text{C}$ . ♦

*example* ►

Solve for  $R_1$  in the equation,  $\frac{1}{R_3} = \frac{1}{R_1} + \frac{1}{R_2}$ , in terms of  $R_2$  and  $R_3$ .

*solution:*

- The first step is to isolate  $R_1$  from the other variables. This may be done by subtracting  $1/R_2$  from both sides to give,  $\frac{1}{R_1} = \frac{1}{R_3} - \frac{1}{R_2}$ .

- The next step is to multiply the terms on the right hand side of the equation by  $R_2/R_2$  and  $R_3/R_3$  respectively, so as to obtain a common denominator of  $R_2R_3$ .

$$\frac{1}{R_1} = \frac{R_2}{R_2 R_3} - \frac{R_3}{R_2 R_3}$$

- Now subtract to get,  $\frac{1}{R_1} = \frac{R_2 - R_3}{R_2 R_3}$ .

- Finally taking the reciprocal gives:  $R_1 = \frac{R_2 R_3}{R_2 - R_3}$ . ♦

*assignment* ►

The Arrhenius equation relates the temperature  $T$  to the rate constant  $k$ , and the energy of activation  $E$ . It may be written as,

$$\log \frac{k_2}{k_1} = \frac{E}{2.3 R} \left[ \frac{1}{T_1} - \frac{1}{T_2} \right]. R \text{ is a constant.}$$

Which of the following is a correct alternative form of the Arrhenius equation?

$$\begin{aligned} \text{A. } \log \frac{k_2}{k_1} &= \frac{E}{2.3 R} \left[ \frac{1}{T_1 - T_2} \right] \\ \text{B. } \log \frac{k_2}{k_1} &= \frac{E R}{2.3} \left[ \frac{1}{T_2 - T_1} \right] \\ \text{C. } \log \frac{k_2}{k_1} &= \frac{E}{2.3} \left[ \frac{T_2 - T_1}{T_1 T_2} \right] \\ \text{D. } \log \frac{k_2}{k_1} &= \frac{E}{2.3 R} \left[ \frac{T_1 - T_2}{T_1 T_2} \right] \end{aligned}$$

*solution:*

$$\left[ \frac{1}{T_1} - \frac{1}{T_2} \right] = \left[ \frac{T_2}{T_1 T_2} - \frac{T_1}{T_1 T_2} \right] = \left[ \frac{T_2 - T_1}{T_1 T_2} \right]$$

Answer C. ♦

### C. Fractions

$1/2 = 0.500$

$1/3 = 0.333$

$1/4 = 0.250$

$1/5 = 0.200$

$1/8 = 0.125$

$1/9 = 0.111$

$1/10 = 0.10$

$1/100 = 0.010$

When working with fractions, if the *numerator* increases by 100, the number *increases* by 100. For example since  $1/3 = 0.333$ , then  $100/3 = 33.3$ . When the *denominator* increases by 100, the number *decreases* by 100. For example since  $1/2 = 0.500$ , so  $1/200 = 0.00500$ .

*example* ►

Solve for  $x$  in the equation:

$$\frac{1}{2} + \frac{1}{5} = x$$

*solution:*

To add these fractions we need a common denominator. The lowest common denominator divisible by both 2 and 5 is 10. So we multiply  $1/2$  by  $5/5$ , and  $1/5$  by  $2/2$  to give:

$$\frac{5}{10} + \frac{2}{10} = \frac{7}{10} \quad \blacklozenge$$

Or we can use the decimal equivalents of the fractions:

$$0.5 + 0.2 = 0.7 \quad \blacklozenge$$

**example ►**Solve for  $x$  in the equation:

$$\frac{\frac{3}{4}}{\frac{5}{2}} = x$$

**solution:**

First we invert the denominator and bring it to the top, to give:

$$\frac{3}{4} \times \frac{2}{5} = x$$

Next we multiply the numerators, and we multiply the denominators, to get  $6/20$ , which may also be expressed as  $3/10$  or  $0.3$ .

$$\frac{6}{20} = \frac{3}{10} = 0.3 \blacklozenge$$

**D. Percent**

If one out of every two applicants are admitted to medical school, we may represent this ratio as the fraction  $1/2$ , as the decimal  $0.5$ , or as a *percent*. **Percent** % is a fraction (or its decimal equivalent), multiplied by 100. Therefore, the percent of applicants accepted would be  $50\%$ .

If the total number of applicants were 30,000, we could find the number of students admitted by multiplying 30,000 by  $0.5$  (or by  $1/2$ ) and get 15,000.

**E. Exponents**

$$(x)^0 = 1$$

$$(x)^{-1} = 1/x$$

$$(x)^{1/2} = \sqrt{x}$$

$$x^a \cdot x^b = x^{a+b}$$

$$(12)^0 = 1$$

$$(2)^{-1} = 1/2$$

$$(16)^{1/2} = 4$$

$$x^2 \cdot x^3 = x^5$$

$$x^a/x^b = x^{a-b}$$

$$x^2/x^3 = x^{-1}$$

$$(x^a)^b = x^{ab}$$

$$(x^2)^5 = x^{10}$$

$$b\sqrt{x^a} = x^{a/b}$$

$$^3\sqrt{x^9} = x^3$$

**F. Logarithms**

A logarithm is a way of expressing a number as the power of 10. The number 100 is equal to  $10^2$ , so the logarithm of 100 is 2. The number 0.01 is equal to  $10^{-2}$ , so the logarithm of 0.01 is -2.

An antilog is a way of returning a logarithm back to its original number. For example, the antilog of 2 is equal to  $10^2$  or 100. The antilog of -2 is equal to  $10^{-2}$  or 0.01.

► To find the antilog of a number, simply make the number an exponent of ten (antilog # =  $10^{\#}$ ).

$$\log(a \cdot b) = \log a + \log b$$

$$\log 6 = \log 2 + \log 3$$

$$\log a/b = \log a - \log b$$

$$\log 2/3 = \log 2 - \log 3$$

$$\log a^b = b \cdot \log a$$

$$\log 2^5 = 5 \cdot \log 2$$

$$\log \sqrt[b]{a} = 1/b \cdot \log a$$

$$\log \sqrt{7} = 1/2 \cdot \log 7 \quad (\text{Recall } \sqrt{\quad} \equiv {}^2\sqrt{\quad})$$

## G. Change versus Proportional Change

The symbol  $\Delta$  is used to represent the **difference** between a final and an initial value of some variable. For example, if the temperature changes from 100 K to 300 K the temperature change  $\Delta T$  is defined as  $T_2 - T_1$ , or  $300 \text{ K} - 100 \text{ K} = 200 \text{ K}$ . Because the size of a Celsius degree is equal to the size of a Kelvin, a change of 200 K is equivalent to a change of  $200^\circ\text{C}$ . A temperature of 200 K, however, is NOT equivalent to a temperature of  $200^\circ\text{C}$ ! The former example is a *change* in temperature, the latter is the temperature.

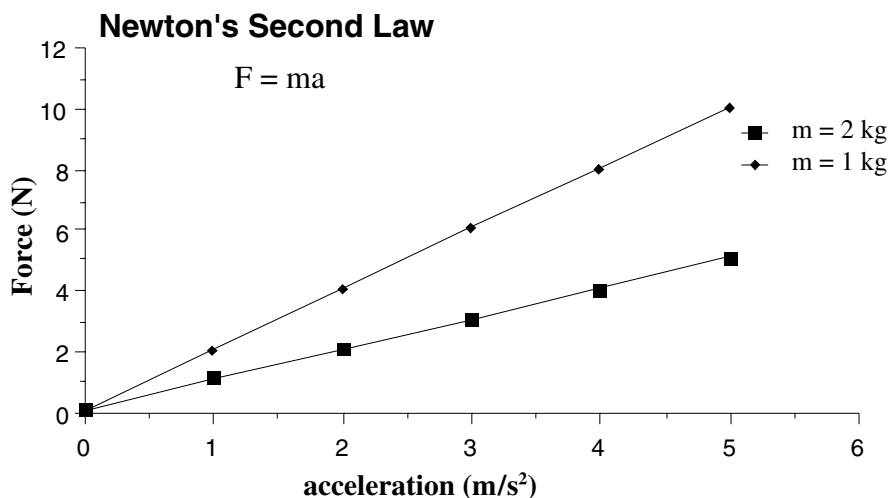
In the example above, the **proportional change** in the temperature is  $T_2/T_1$ , or  $300 \text{ K}/100 \text{ K} = 3$ . This indicates that the temperature has tripled. Alternatively if we multiply by 100, we may say that there has been a 300% increase in the temperature. This calculation is only valid in Kelvin, because only Kelvin temperatures are directly proportional to each other (See Section 2.1K and Chapter#■)

► Only the Kelvin scale may be used to determine proportional changes in temperature.

A temperature increase from 100 K to 300 K, or a temperature change from 300 K to 900 K, both represent a three fold increase. These two sets of temperature changes are said to be **proportional** to each other, since they both represent the same proportional change,  $T_2/T_1 = 3$ .

## H. Direct Proportionality & Slope

A pair of variables are said to be **directly proportional** if, when multiplied by a proportionality factor, they can be set equal to each other. For example, in Newton's second law,  $F = ma$ , the variables  $F$  and  $a$  are directly proportional. In this example  $m$  is the proportionality factor. If  $m$  remains constant,  $a$  will double as  $F$  doubles. The relationship between  $F$  and  $a$  is depicted below for both a 1 and a 2 kg mass. Although in both cases  $a$  doubles as  $F$  doubles, note that in the case of the 2 kg mass, twice the force is required to achieve the same acceleration. This difference is reflected in the different slopes shown in Figure 2.1.



**Figure 2.1** A direct proportionality with a positive slope.

■ The **slope** of a line is defined as the change in the vertical axis, divided by the change in the horizontal axis.

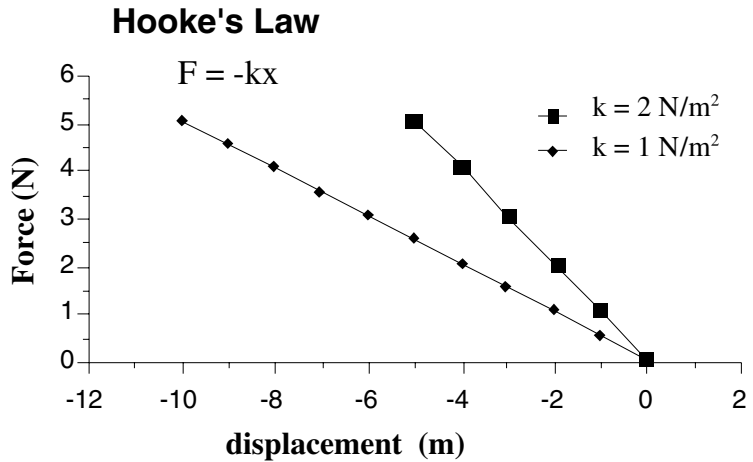
$$\text{Slope} = \Delta y / \Delta x$$

**Equation 2.1**

In Figure 2.1 the slope for the 2 kg mass is 2 N/m/s<sup>2</sup>, while that for the 1 kg mass is 1 N/m/s<sup>2</sup>.

Hook's Law,  $F = -kx$ , relates the Force  $F$ , of a spring to its displacement from its equilibrium position,  $x$ .

The negative sign in this equation indicates that the direction of the displacement and the resulting force have opposite orientations. The relationship between  $F$  and  $x$  in Hooke's law is similar to the relationship between  $F$  and  $a$  in Newton's law. Note, however, that Hooke's law results in a negative slope.

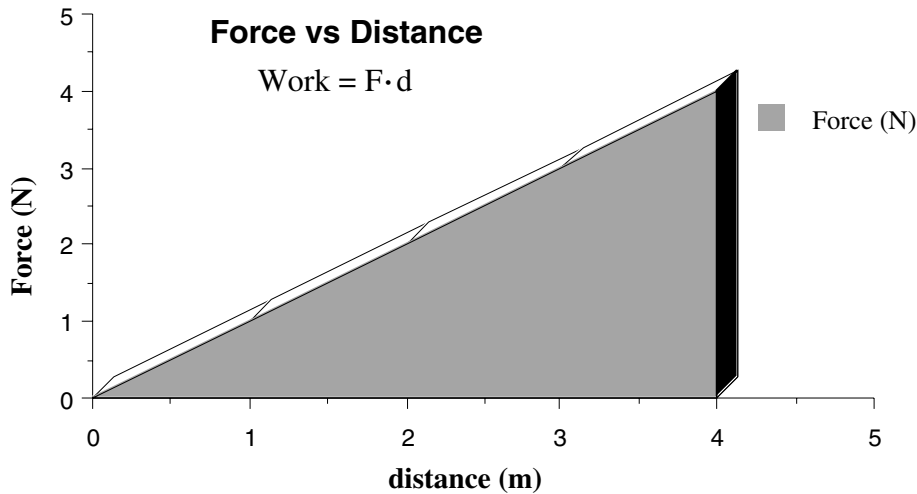


**Figure 2.2** A direct proportionality with a negative slope.

*Some physics texts do not use the negative sign in Hooke's law. They redefine  $F$  to refer to the force that is applied to the spring, rather than the force which results from the spring. In this case the plots of Newton's and Hooke's law will both have positive slopes.*

### I. Area under a curve

In Figure 2-3 of force versus distance, the total work may be calculated by finding the area under the curve. This is expressed by the equation  $Work = Force \times Distance$ .

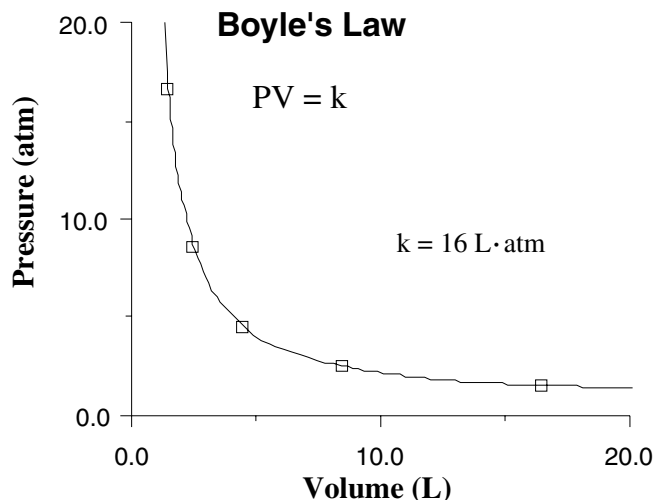


**Figure 2.3** Work as a function of the area under a curve.

The area under the curve is depicted by the triangle above. The area of a triangle is given by one half its base times its height. The total work, therefore, is  $\frac{1}{2}(4 \text{ m})(4 \text{ N}) = 8 \text{ N} \cdot \text{m}$ .

## J. Inverse Proportionality

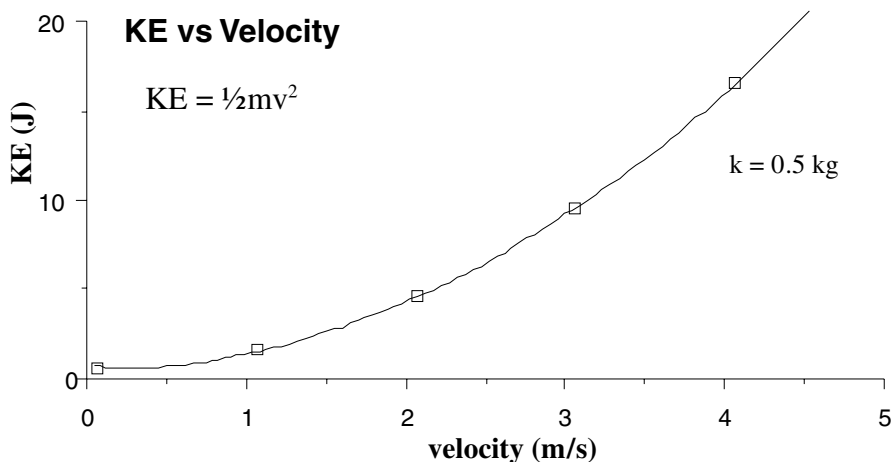
Boyle's law states that for a given quantity of an ideal gas at a constant temperature, the pressure multiplied the volume will be constant.  $PV = k$  or  $P = k/V$  or  $P = k(V)^{-1}$ . Variables related in this manner are said to be inversely proportional. When the pressure doubles the volume will half. If the pressure were to drop by a quarter, the volume would quadruple. In other words, when one variable goes down, the other goes up by the same proportional amount.



**Figure 2.4** An inverse proportionality.

Mathematical relationships, like Hooke's law (previously discussed) which take the form,  $y = k(x)^1$ , are first order equations and will always produce a straight line. Boyle's law, on the other hand, takes the form  $y = k(x)^{-1}$ . Its order is negative one. See Section 2.4 for additional information concerning the order of an equation.

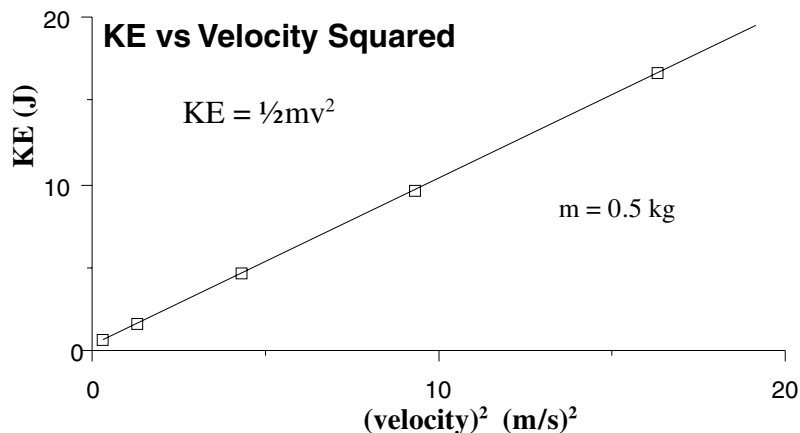
■ Only first order equations produce linear plots, when one variable is plotted directly against another.



**Figure 2.5**

The equation for translational kinetic energy,  $KE = \frac{1}{2}mv^2$ , is second order with respect to velocity. When mass is held constant, the kinetic energy quadruples as the velocity doubles. If the velocity were to drop by one fifth, the kinetic energy would drop by to one–twenty fifth. This relationship is represented in the exponential curve depicted in Figure 2-5.





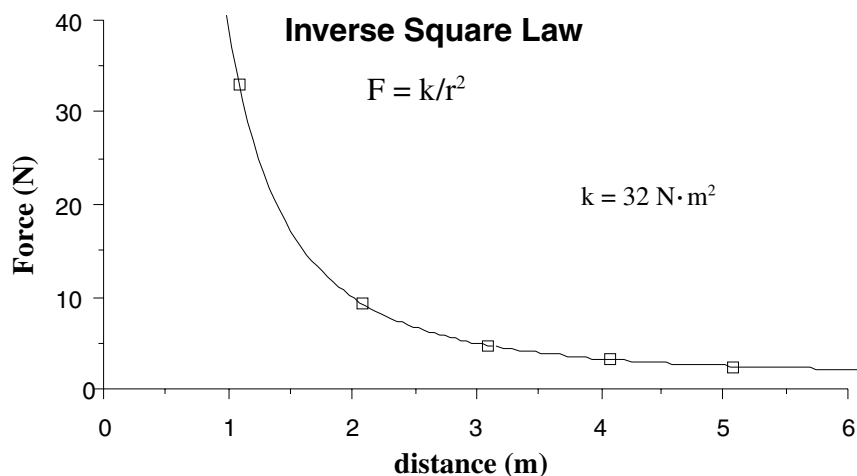
**Figure 2.6**

Although KE is not proportional to the velocity, it *is* proportional to the *square* of the velocity. This relationship is depicted in Figure 2.6. Note that we have obtained a linear plot by placing the *square* of the velocity on the horizontal axis.

A common mathematical form on the MCAT is the **inverse square**. Note that in the two equations below, the force  $F$  is inversely proportional to the distance squared  $r^2$ .

$$F_{\text{gravity}} = \frac{Gm_1m_2}{r^2} \quad F_{\text{electrostatic}} = \frac{kq_1q_2}{r^2}$$

The relationship between force and distance in the Law of Universal Gravitation, and Coulomb's Law shown respectively above, can be captured by the expression  $F \propto k/r^2$ . Where  $\propto$  is the symbol for proportionality, and  $k$  is the proportionality constant. When the distance  $r$  between two masses, or two charges, doubles, the force acting between them will drop to one fourth of its previous value. This inverse square relationship is depicted in Figure 2.7.



**Figure 2.7**

## 2.2 Units of Measurement

### A. Metric System Prefixes

pico $p = 10^{-12}$	nano $n = 10^{-9}$	micro $\mu = 10^{-6}$	milli $m = 10^{-3}$
centi $c = 10^{-2}$	deci $d = 10^{-1}$	kilo $k = 10^3$	mega $M = 10^6$
giga $G = 10^9$	tera $T = 10^{12}$		

### B. SI System

Quantity (symbol)	Name of Unit	Unit's Symbol	Description
Temperature (T)	kelvin	K	base unit
Quantity of matter (n)	mole	mol	base unit
Mass (m)	kilogram	kg	base unit
Displacement* (d or s)	meter	m	base unit
Distance (d or s)	<i>same as above</i>		
Angular Displacement	radian	rad	supplementary unit
Time (t)	second	s	base unit
Velocity* (v)	—	none (m/s)	Displacement/Time
Acceleration* (a)	—	none (m/s <sup>2</sup> )	Displacement/(Time) <sup>2</sup>
Momentum* (p)	—	— (kg·m/s)	(Mass)(Velocity)
Force* (F)	newton	N (kg·m/s <sup>2</sup> )	(Mass)(Acceleration)
Pressure (P)	Pascal	Pa (N/m <sup>2</sup> )	Force/Area
Torque* ( $\tau$ )	—	— (N·m)	—
Energy (E)	joule	J (N·m)	(Force)(Displacement)
Work (W)	<i>Same as above.</i>		
Power (P)	watt	W (J/s)	Energy/Time
Frequency (f or $\nu$ )	hertz	Hz (s <sup>-1</sup> )	Oscillations /Time
Radioactivity Activity (—)	becquerel	Bq (s <sup>-1</sup> )	Disintegrations/Time
Electrical Charge (q)	coulomb	C	base unit
Elect. Potential (E)	volt	V (J/C)	Energy/Charge
Electromotive force (emf)	<i>Same as above.</i>		
Elect. Current (I)	ampere	A (C/s)	Charge/Time
Elect. Resistance (R)	ohm	$\Omega$ (V/A)	Potential/Current
Capacitance (C)	farad	F (C/V)	Charge/Elect. Potential

Electric Field* (E)	—	— (N/C)	Force/Charge
Magnetic Field* (B)	tesla	T (N·s/C·m)	Force/Charge·Velocity
Magnetic Flux* (F)	weber	W (T·m <sup>2</sup> )	(Magnetic Field)(Area)

\*Vector quantities. See section 2.4D.

*The symbols listed above are in common use, but may vary somewhat from one textbook to another.*

### C. Conversion Factors & Constants

$$1000 \text{ g} = 1 \text{ kg}$$

$$1000 \text{ mL} = 1 \text{ L}$$

$$760 \text{ torr} = 760 \text{ mm Hg} = 1 \text{ atm}$$

$$\text{Avogadro's number} \approx 6 \times 10^{23} \text{ particles /mole}$$

$$\text{Density of H}_2\text{O} \approx 1 \text{ g/mL}$$

$$\text{Charge on an electron} = -1.6 \times 10^{-19} \text{ C}$$

$$1 \text{ angstrom (1\AA)} = 10^{-10} \text{ m}$$

$$^\circ\text{C} = \text{Kelvin degrees} - 273$$

$$1.00 \text{ calorie} \approx 4.18 \text{ joules}$$

$$^* 1 \text{ m}^3 \times \left(\frac{100 \text{ cm}}{\text{m}}\right)^3 \times \frac{\text{L}}{1000 \text{ cm}^3} = 1000 \text{ L}$$

$$1 \text{ mL} = 1 \text{ cc (cubic centimeter)}$$

$$1000 \text{ L} = 1 \text{ m}^3 \text{ }^*$$

$$1 \text{ atm} \approx 10^5 \text{ Pa}$$

$$6 \times 10^{23} \text{ atomic mass units (amu)} \approx 1.0 \text{ gram}$$

$$\text{Density of H}_2\text{O} \approx 1000 \text{ kg/m}^3$$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

$$\text{Acceleration due to gravity} = 9.8 \text{ m/s}^2 \approx 10 \text{ m/s}^2$$

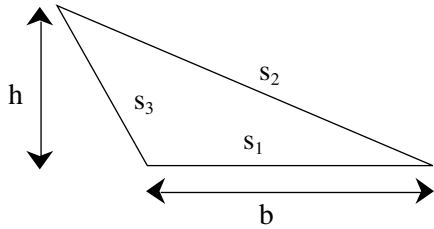
$$\text{Speed of light in vacuum} \approx 3 \times 10^8 \text{ m/s}$$

$$\sqrt{2} \approx 1.4$$

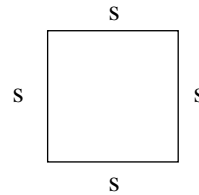
## 2.3 Geometry

### A. Geometric Formulas

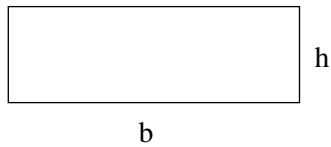
(b = base, h = height, l = length, r = radius, s = side, w = width)



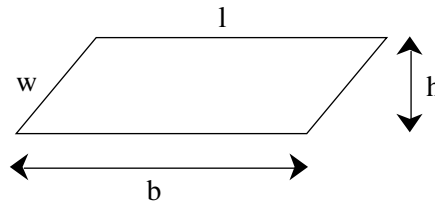
**TRIANGLE**  
 Perimeter =  $s_1 + s_2 + s_3$   
 Area =  $\frac{1}{2}(b \times h)$



**SQUARE**  
 Perimeter =  $4s$   
 Area =  $s \times s$



**RECTANGLE**  
 Perimeter =  $2(b + h)$   
 Area =  $b \times h$



**PARALLELOGRAM**  
 Perimeter =  $2(l + w)$   
 Area =  $b \times h$   
 (Note that  $w > h$ )

**Figure 2.8**

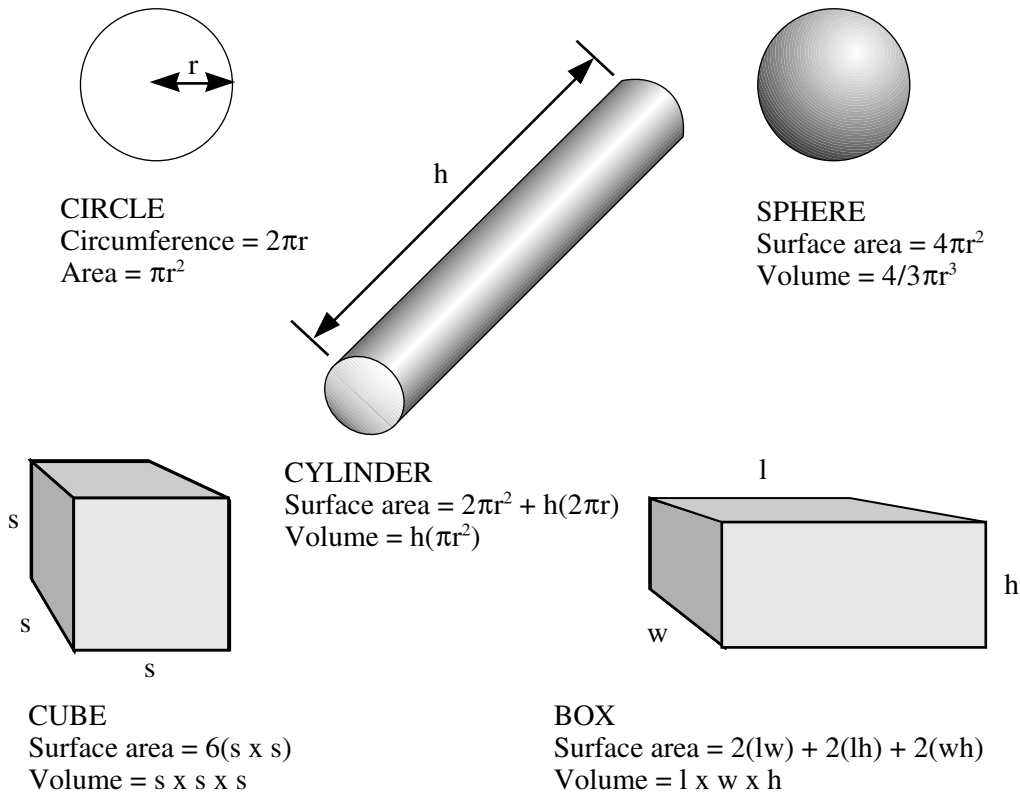


Figure 2.9

## B. Angles

■ A right angle is any  $90^\circ$  angle. An acute angle is any angle less than  $90^\circ$ . An obtuse angle is any angle greater than  $90^\circ$ .

■ When two parallel lines are intersected as in Figure 2.10 (b), the angles  $\theta + \phi$  will equal  $180^\circ$ . The acute angle  $\phi$  is always adjacent to the obtuse angle  $\theta$ . Also note that  $\phi$  is always diagonal to  $\theta$ , and  $\theta$  is always diagonal to  $\phi$ .

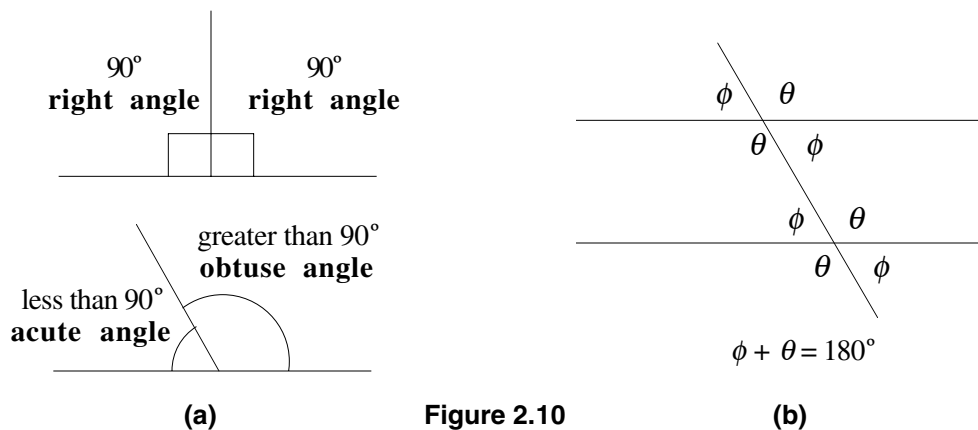
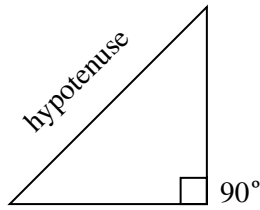


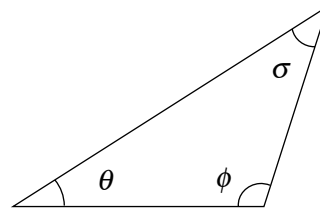
Figure 2.10

### C. Triangles

- A right triangle is any triangle with a  $90^\circ$  angle. The side opposite the  $90^\circ$  angle is called the **hypotenuse**.
- The sum of the three angles in any triangle are equal to  $180^\circ$ .



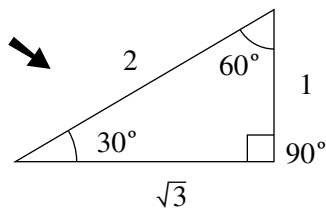
A right triangle



$$\theta + \phi + \sigma = 180^\circ$$

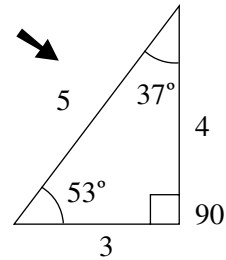
**Figure 2.11**

- The largest angle opposes the longest side. The smallest angle opposes the shortest side.



$$90^\circ > 60^\circ > 30^\circ$$

$$2 > \sqrt{3} > 1$$

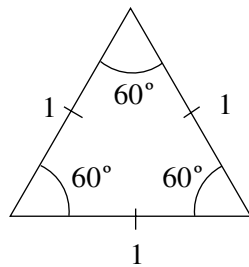


$$90^\circ > 53^\circ > 37^\circ$$

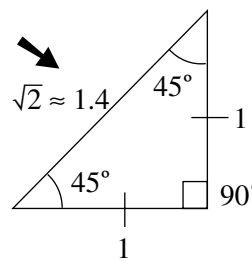
$$5 > 4 > 3$$

**Figure 2.12**

- An **equilateral triangle** has three  $60^\circ$  angles and therefore, has three sides of equal length.
- An **isosceles triangle** has two equal angles, and therefore, has two sides of equal length.



An equilateral triangle

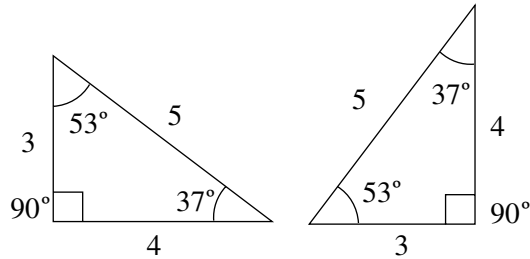


An isosceles right triangle

**Figure 2.13**

➤ The triangles marked by an arrow in Figure 2.13, frequently appear on the MCAT

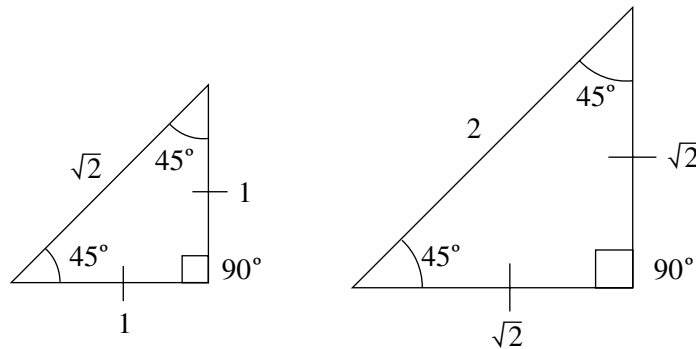
■ If it is known that two of a triangle's sides are equal to two sides of another triangle, then these triangles are **congruent**. Congruent triangles are identical two each other. They share three of the same angles, and three sides of the same lengths.



Congruent Triangles

Figure 2.14

■ If two triangles are not congruent, but it is known that they share two angles, then these triangles are **similar**. Similar triangles are identical to each other in proportion, but are different in size. They share three of the same angles, but their side lengths, although proportional, are not equal.



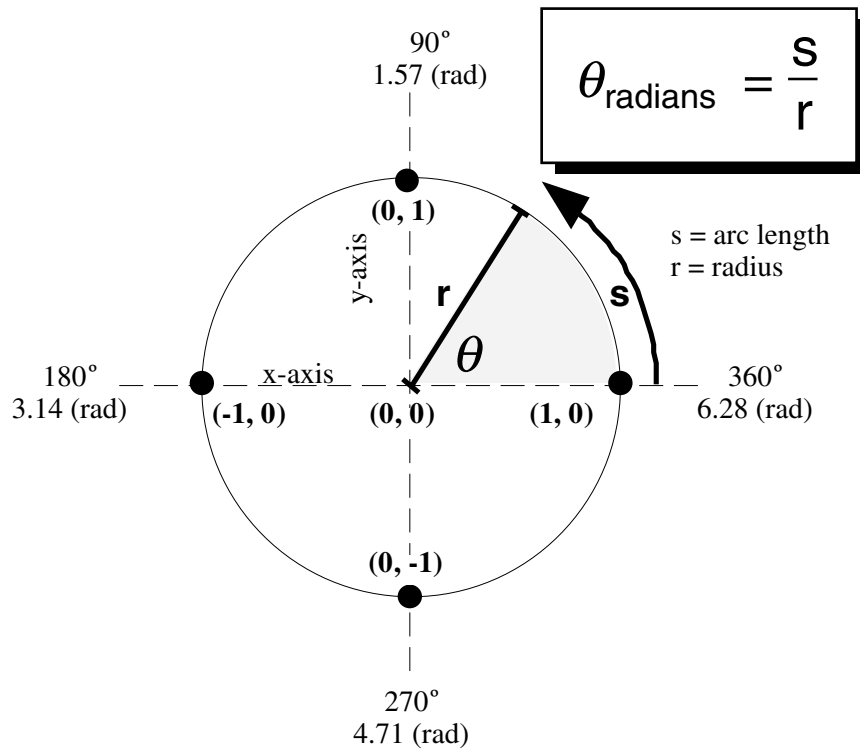
Similar Triangles

Figure 2.15

## 2.4 Trigonometry

### A. Degrees and Radians

The formula  $x^2 + y^2 = r^2$ , describes a circle with a radius equal to  $r$ , and its center on the origin of the  $x$  and  $y$  axes. The circle shown in Figure 2.16 has a radius equal to *one*, and therefore is called a *unit circle*.



**Figure 2.17**

If we start at the point labeled  $(1, 0)$  and make one complete revolution around the circle, the arc length  $s$  will be the circumference of the circle and will equal to  $2\pi r$ , where  $r = 1$  and  $\pi = 3.14$ .

The angular displacement  $\theta$  is equal to  $360^\circ$  for one complete counterclockwise revolution and  $-360^\circ$  for a clockwise rotation. Angular displacement may also be described in radians. Lets look at the angle  $\theta$  depicted by the shaded wedge within the unit circle shown above. The angular displacement in radians is found by dividing the arc length  $s$  by the radius  $r$ . The result is a unit-less number. Since  $s$  equals  $2\pi r$  for one complete revolution, the number of radians per revolution is  $2\pi r/r$ . This equals  $2\pi$  radians (or  $2 \times 3.14$  radians =  $6.28$  radians). By dividing  $6.28$  radians into  $360^\circ$  we find that one radian is equal to  $57.3^\circ$ .

$$1 \text{ rev} = 2\pi r = 360^\circ$$

$$\theta_{\text{radians}} = \frac{s}{r}$$

$$\frac{2\pi r}{\text{rev}} \times \frac{1}{r} = \frac{2\pi(\text{rad})}{\text{rev}} = \frac{6.28(\text{rad})}{360^\circ}$$

$$\frac{360^\circ}{6.28(\text{rad})} = \frac{57.3^\circ}{(\text{rad})}$$

**Figure 2.16**

Note that if the radius of the circle were to double, the arc length required to complete one revolution would also double, but the angular displacement would remain the same.



## B. The Pythagorean Theorem

Recall that a right triangle has a  $90^\circ$  angle opposite its longest side called the hypotenuse. Its other two sides are referred to as legs.

**The Pythagorean theorem** states that the square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of its legs.

For example if the length of  $a$  is 0.6, and the length of  $b$  is 0.8 then the length of the hypotenuse  $c$  is given by:  $c = \sqrt{a^2 + b^2}$ . So,

$$c = \sqrt{(0.36) + (0.64)} = 1.00$$

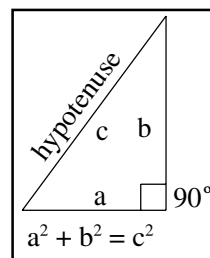


Figure 2.18

Note that the Pythagorean theorem, and the equation for a circle are closely related. The points that define the unit circle, are given by all sets of  $x$  and  $y$  values that satisfy the condition  $x^2 + y^2 = 1^2$ . This equation also describes all right triangles with leg lengths of  $x$  and  $y$  and a hypotenuse equal to one. As  $\theta$  increases from the starting point (indicated by an arrow), the leg labeled  $x$  will decrease in length, while the leg labeled  $y$  will increase. The trigonometric functions sine, cosine and tangent quantitatively describe the changes in  $x$ ,  $y$ , and  $\theta$

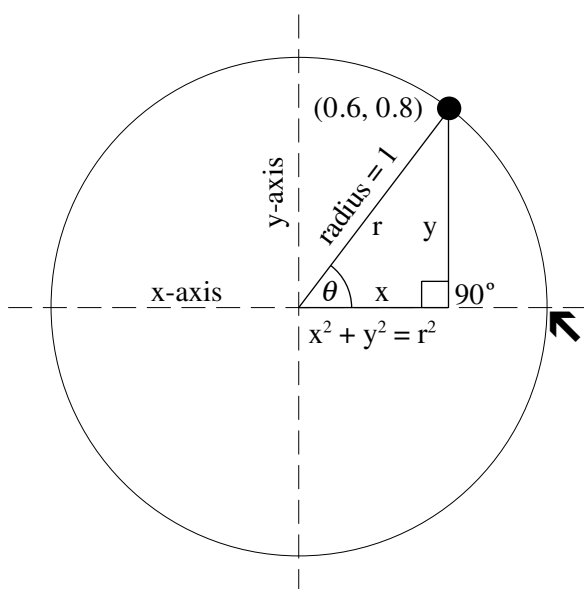


Figure 2.19

## C. Sine, Cosine, and Tangent

The definitions of the sine, cosine, and tangent functions are given in Figure 2.20.

If side  $a$  is equal to 3, and side  $o$  is equal to 4, then by applying the Pythagorean theorem we may calculate the hypotenuse to be equal to 5.

The value for  $\sin \theta$  would then equal  $4/5$ . The value for  $\cos \theta$  would be  $3/5$ , and  $\tan \theta$  would be  $4/3$ .

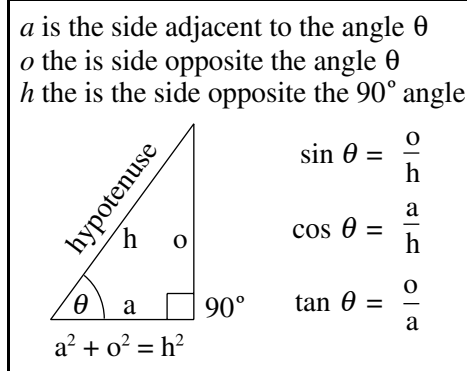


Figure 2.20

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### SOH CAH TOA

SOH:  $\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$

CAH:  $\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$

TOA:  $\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}}$

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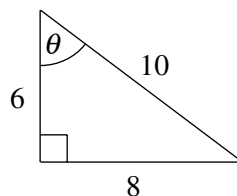


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**assignment ►**

Which of the following is true with regard to the triangle shown below?

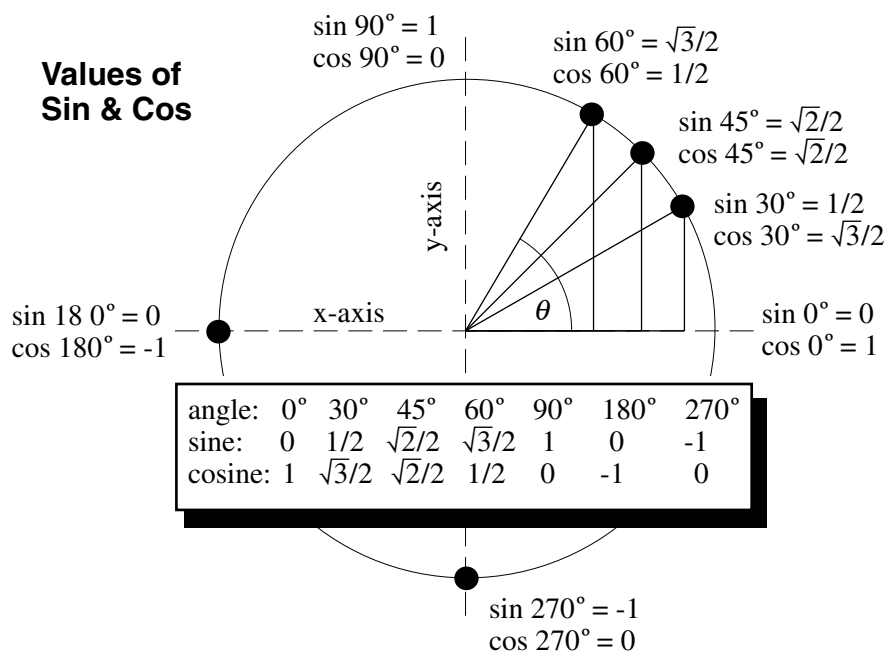
- A.  $\sin \theta = 3/5$
- B.  $\sin \theta = 4/5$
- C.  $\sin \theta = 4/3$
- D.  $\sin \theta = 3/4$



**solution:**

Answer **B**. Since  $\sin \theta$  is defined as the side opposite  $\theta$  divided by the hypotenuse, the correct answer is  $8/10$  or  $4/5$ . *If you choose answer A, remember that you cannot assume that the vertical leg will always be related to the sine function.*

In Figure 2.21, the three right triangles inscribed in a unit circle are used to illustrate important values of the sine and cosine functions.



**Figure 2.21**

■ Note that the length of each vertical leg is equal to  $\sin \theta$ , the length of each horizontal leg equals  $\cos \theta$ , and  $\tan \theta$  is equal to the slope of the hypotenuse ( $\Delta y / \Delta x$ ). Since  $\Delta y = \sin \theta$ , and  $\Delta x = \cos \theta$ , it follows that:

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

**Equation 2.2**

- Note that at  $45^\circ$ ,  $\sin \theta$  is equal to  $\cos \theta$ , so the tangent of  $45^\circ$  is equal to one.

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**A GOOD “ONE” TO REMEMBER**

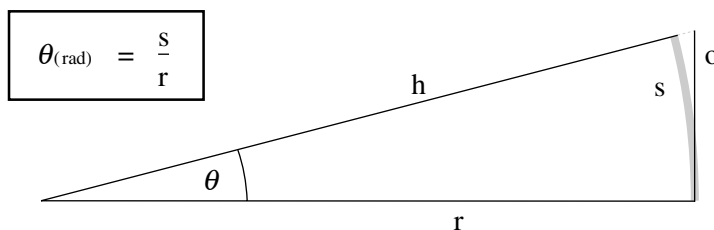
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$$\sin 90^\circ = \cos 0^\circ = \tan 45^\circ = 1$$

Referring to Figure 2.22, for small angles where  $r$  is much larger than  $o$ , the angular displacement  $\theta$  can be approximated by Equation 2.3. *Note that  $\theta$  will be in radians.*



**Figure 2.22**

*For small angles:*

$$\theta_{(\text{rad})} \approx \frac{o}{r}$$

**Equation 2.3**

Equation 2.3 is valid for small angles because  $o \approx s$ . Any angle with  $r$  more than twice as great as  $o$  may be approximated by this method. Also note that since  $r \approx h$ ,  $\sin \theta \approx \tan \theta$ .

## D. Vectors

- Physical quantities may be classified as either scalar or vector quantities.
- A scalar quantity has magnitude, but no direction. Mass and volume are examples of scalar quantities.
- Vector quantities have both magnitude and direction. Force and acceleration are examples of vector quantities.

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**VECTORS**

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A vector's position is generally irrelevant; only its magnitude and direction are important.

- An arrow drawn above a variable *may* be used to signify a vector quantity. In the Equation 2.4,  $F$  and  $a$  are labeled as vector quantities.

$$\vec{F} = m\vec{a}$$

**Equation 2.4**

- When two scalars are added, subtracted, multiplied or divided, the result will always be a scalar.

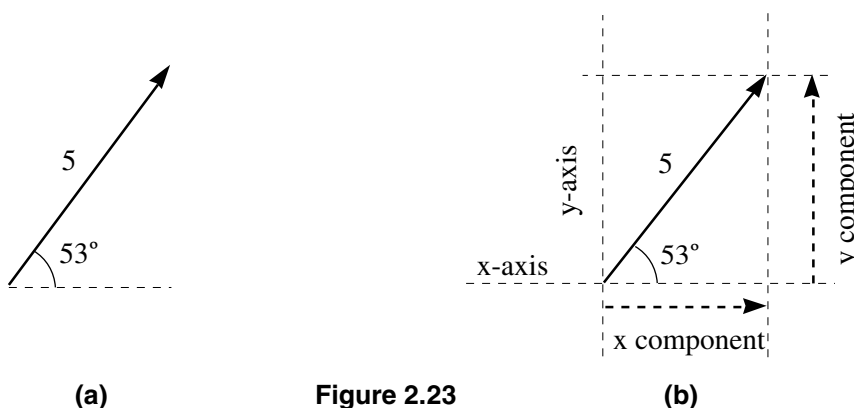
■ When a one vector is added or subtracted from another vector, the result will also be a vector. Vector addition and subtraction are demonstrated below.

■ When a vector is multiplied or divided by a scalar, the result will be a new vector with the same direction but a different magnitude.

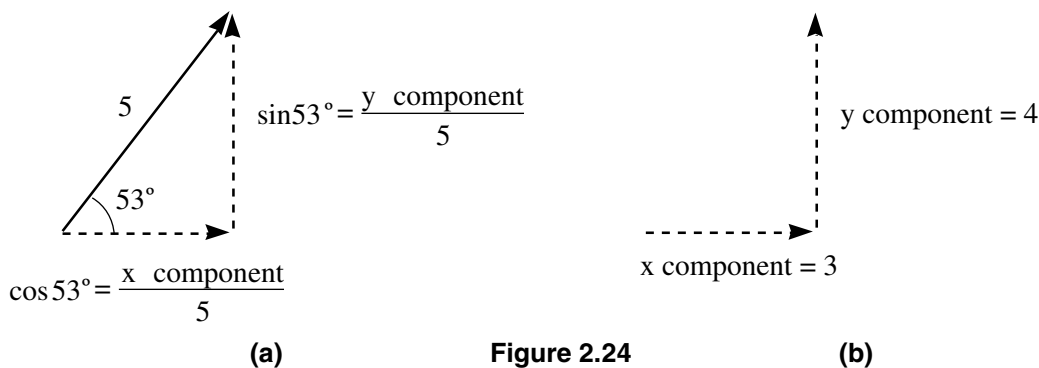
■ When a vector is multiplied or divided by another vector, a new vector will result. Vector multiplication is dealt with on the MCAT only by application of the right hand rule. The right hand rule is covered in Chapter#■.

■ A vector may be represented graphically by an arrow as depicted in Figure 2.23(a). The arrow's length represents the vector's magnitude. The orientation of the arrow indicates the vectors direction. Figure 2.23(a) depicts a vector with a magnitude of five, directed  $53^\circ$  above the horizontal.

■ A vector which does not lie on either the x or y axis may be resolved into its x and y components by the application of the cosine and sine functions. The x component of a vector is equivalent to its length when measured along the x axis. Its y component is its height measured along the y axis. This relationship is demonstrated in Figure 2.23(b).



The application of the cosine and sine functions to determine the x and y components of a vector quantity is illustrated below.

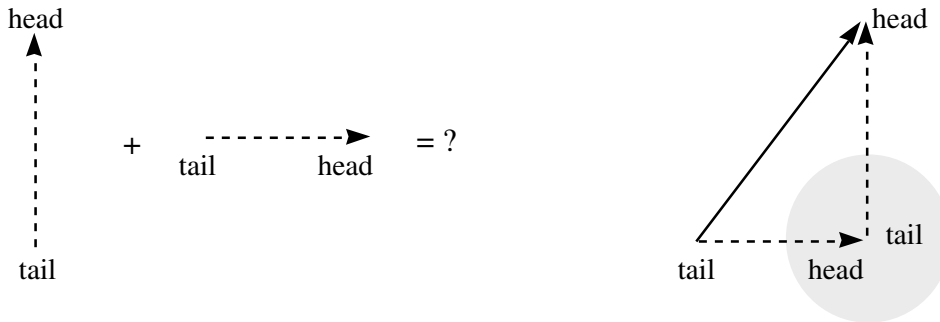


■ Vectors may be added or subtracted by use of a graphic or a mathematical procedure.

■ **In the graphic approach**, shown in Figure 2.25, two vectors will be added. When using this method it is important to remember that a vector representation may be moved about freely, as long as its length and orientation are not changed.

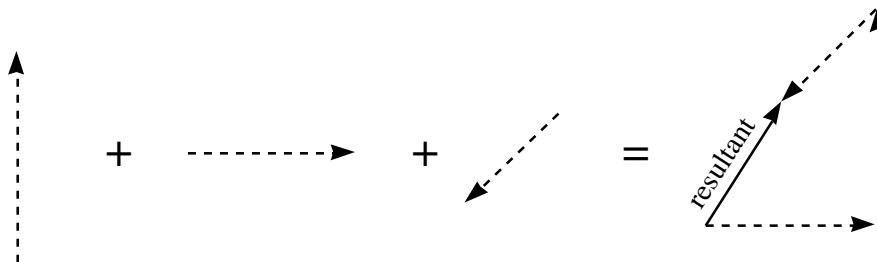
The first step in adding two vectors is to place the *tail* (the end opposite the arrow *head*) of one vector on to the head of the other. An arrow is drawn from the free tail of one vector to the head

of the other, as depicted in Figure 2.25(b). The new arrow represents the vector resulting from the addition.



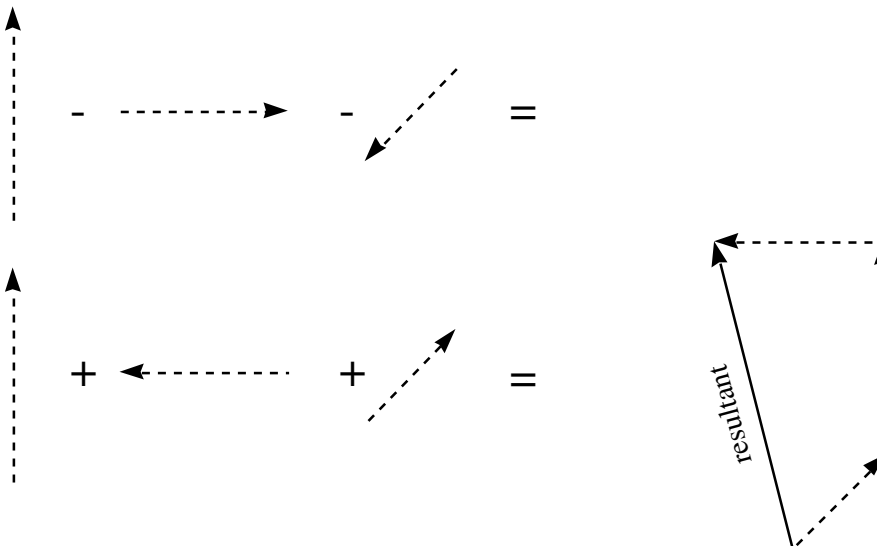
**Figure 2.25**

Any number of vectors may be added together by the use of the “tail to head” method. *The sequence in which vectors are added will not effect the result.* Below three vectors are added.



**Figure 2.26**

To subtract vectors, the vectors which bear the negative signs are reversed in direction and then added in the same manner as previously outlined. An example follows.

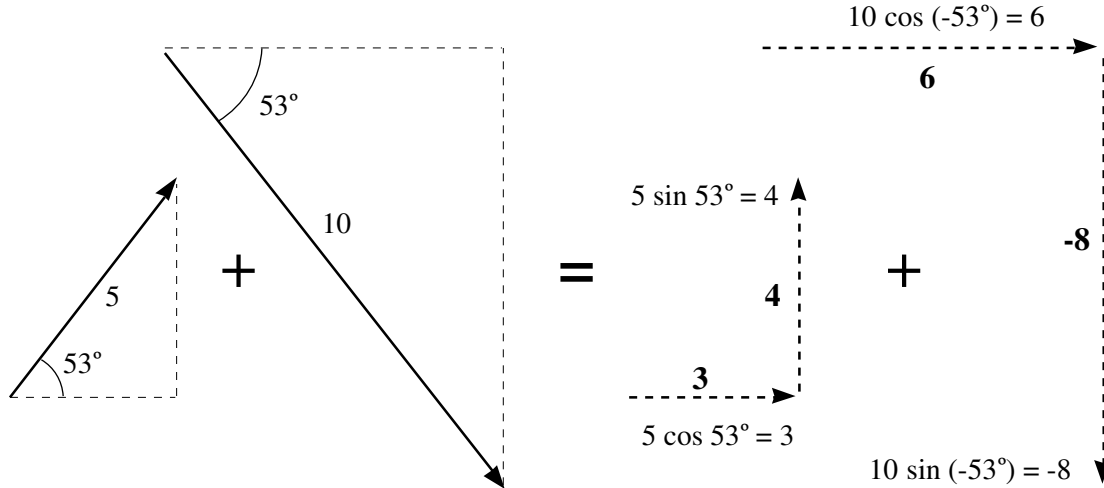


**Figure 2.27**

■ **To add vectors mathematically**, each vector is broken down into its x and y components. The x and y components are then added together. The Pythagorean theorem and the tangent function are then used to obtain the resultant magnitude and direction. An example follows.

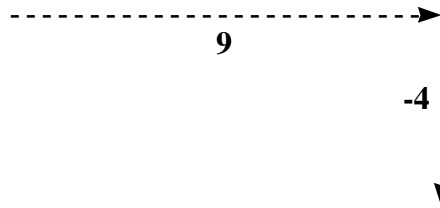
► In the following example vectors are assigned a negative value if they are directed downward or to the left. Angles are assigned a negative value if they are directed downward (i.e., a clockwise rotation of less than 180° with respect to the positive x axis.)

**1) Resolve each vector into x and y components.**



**2) Add the x and y components independently.**

x components =  $3 + 6 = 9$   
 y components =  $4 + (-8) = -4$



**3) Use the Pythagorean theorem to get the magnetude, and the tangent function to get the angle.**

magnitude =  $\sqrt{(9)^2 + (-4)^2}$   
 $= \sqrt{97} \approx \sqrt{100}$

The magnitude is just less than 10. ♦

$\tan \theta = \frac{-4}{9}$

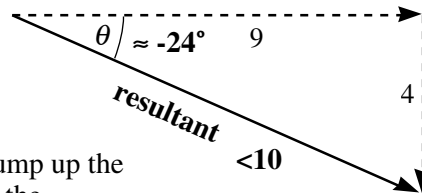
$\theta(\text{rad}) \approx \frac{-4}{9}$

$\theta(\text{deg}) \approx \frac{-4}{9} \text{ (rad)} \times \frac{573^\circ}{\text{(rad)}}$

$\theta(\text{deg}) \approx \frac{-4}{10} \text{ (rad)} \times \frac{60^\circ}{\text{(rad)}}$

Ok now lets bump up the numerator and the denominator to make the math ridiculously easy!

$\theta \approx -24^\circ$  ♦ (The negative sign indicates a clockwise rotation.)



The method for mathematically subtracting vectors involves the same steps as outlined above. In step two, however, one must subtract rather than add.