Thanks for downloading THE MCAT *EXCELERATOR™*. Although they are only rough drafts, I hope these chapters will help you out on the MCAT. I'd love to hear any comments, corrections or suggestions you may have.
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Linear Momentum

6.1 Momentum and Impulse

Momentum

Linear momentum, *p* (a.k.a. momentum) is defined as the mass of a body muliplied by its velocity. Momentum is a vector quantity with the SI unit kg·m/s.

$$\boxed{p = mv}$$

Equation 6.1

Impulse

In order for a change in momentum to occur a net force must be applied to a body for a given period of time. The average applied force multiplied its duration is called **impulse,** which equals the change in momentum. Impulse is a vector quantity which has the same direction as the applied force. The SI unit of impulse is N·s.

$$\boxed{\text{impulse} = \bar{F}\Delta t = \Delta p}$$

Equation 6.2

assignment ►

In the absence of any outside forces a ball traveling horizontally to the left at 5 m/s stricks a wall and is in contact with the wall for 0.10 s. The ball then rebounds to the right at 10 m/s along the same path it initially followed. What is the implulse? What is the magnetude and direction of the average force exerted on the ball by the wall?

solution:

Impulse = Δp, Impulse = $p - p_o$, Impulse = $mv - mv_o$,

Impulse = $m[v - v_o]$, Impulse = $5[\ 10 - \{-10\}] = 100$ N·s. ◆

Notice that $v = +10$ m/s because it is noving to the right, and $v_o = -10$ m/s because it is moving to the left.

The magnetude of the average force exserted on the ball = $\Delta p/\Delta t$. Or $100/0.10 = 1000$ N. The ball and the wall exert equal and opposite forces on each other. These are normal forces oriented perpendicular and away from the two surfaces involved. Since the wall faces to the right, the average force on the ball is 1000 N directed to the right. ◆

6.2 Conservarion of Momentum

When a system is isolated from any externel net force, the total momentum of that system will be constant. This is known as the **principle of conservation of momentum.** Although this principle applies to bodies interacting in one, two, and three dimensions, only one and two dimensional examples are included on the MCAT.

Conservation in One Dimenstion

assignment ►

Two ice skaters, initially at rest push off each other. If one skater whose mass is 60 kg has a velocity of 2.0 m/s what is the velocity of the other skater whose mass is 40 kg?

solution:

Intially the skaters are at rest so the momentum of the system is zero. By application of the principle of conservation of momentum the final momentum must be equal to the initial momentum:

$p = p_o$, $m_1v_1 + m_2v_2 = 0$, $(60)(2.0) + (40)v_2 = 0$,

$v_2 = -30$ m/s. ◆

Although the actual sign associated with each velocity vector is not irrelavent here, the fact that the signs are opposite indicates that the skaters are moving in opposite directions.

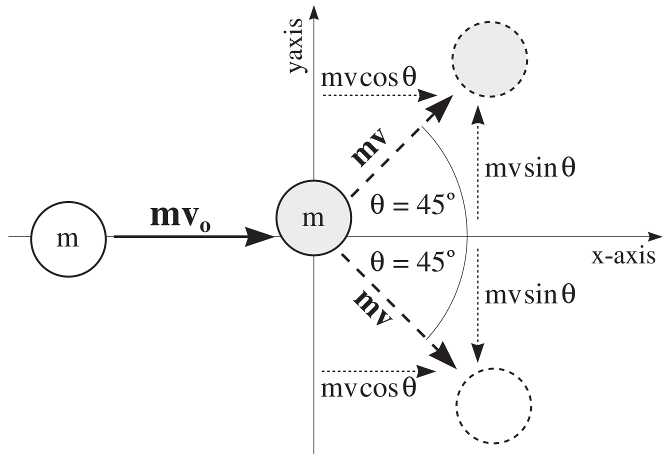
Conservation in Two Dimensions

example ►

A ball of mass *m* moving along the positive x-axis at velocity *v* strikes a glancing blow to a ball of the same mass which is at rest. Each ball then moves away from the x-axis at equal speeds, but in different directions. One ball moves at +45°, while the other moves at –45°. What is the ratio of the first ball's initial velocity to its final velocity?

solution:

In this example we have both the x and the y dimensions to consider. Since this system no velocity component in the y direction, the total momentum in the y dimention will always be zero. In the diagram below the intial momentum vector is shown to lie on the x-axis and is equal to mv_o. The final momentums of each ball are labeled mv. The horizontal componet of each ball's momentum is $mv\cos\theta$, and the vertical component is $mv\sin\theta$.



Ignoring the momentum in the y direction and setting $p_x = p_o$ we get,

$$mv\cos\theta + mv\cos\theta = mv_o,$$

$$2\not{m}v\cos\theta = \not{m}v_o, \qquad v_o/v = 2\cdot\cos 45°, \qquad v_o/v = 2\cdot\sqrt{2}/2$$

$$v_o/v = \sqrt{2} \approx 1.4. \ ◆$$

6.3 Elastic and Inelastic Colisions

Elastic Colisions

Elastic colisions are impacts which occur without any change in the total kinetic energy of the system. For these collisions $\Delta KE = 0$ and $\Delta p = 0$.

assignment ►

Determine if the collision in the previous example was elastic.

solution:

If $KE - KE_o = 0$ the collision was elastic. . Recall that KE is a scalar so we may simply subtract without the use of sine or cosine, and that we found in the previous example that $v_o = \sqrt{2}\,v$

$$\Delta KE = 2(\tfrac{1}{2}mv^2) - \tfrac{1}{2}mv_o^2, \qquad \Delta KE = mv^2 - \tfrac{1}{2}m(\sqrt{2}\,v)^2,$$

$$\Delta KE = mv^2 - mv^2, \qquad \Delta KE = 0, \therefore \text{ the collision was elastic. } ◆$$

Inelastic Colisions

Inelastic colisions occur when the final kinetic energy is less than the initial kinetic energy. For these colisions $\Delta KE < 0$ and $\Delta p = 0$. During a colision kinetic energy is transformed into elastic potential energy and then back into kinetic energy. During this process some mechanical energy is converted into heat.

assignment ►

In the absence of air resistance a 1.0 kg ball is dropped from 2.0 meters but rebounds only to half its original height. How many Joules of heat were released as a result of this inelastic colision? Let $g = 10\ m/s^2$.

solution:

It should be clear that this is indeed an inelastic collision otherwise the ball would have returned back to its original height. As the ball drops gravitational PE is converted into KE, and as the ball strikes the floor the KE is converted into elastic PE. Some heat is produced, and then the whole process reverses. What is important is that quantity of heat produced is equal to ΔKE, which is equal to $\Delta PE_{gravitational}$.

$\Delta PE = mg(h - h_o)$, $\Delta PE = (1)(10)(2 - 1) = 10$ J of heat. ◆

Fluids and Solids

Fluids are matterials which have the ability to flow. *Both gasses and liquids are clasified as fluids.* **Solids** are characterized by their rigidity. While gases are easily compressable, liquids and solids are not.

7.1 Density and Pressure

Density

Density ρ is the mass of a substance divided by its volume V.

$$\rho = \frac{\text{mass}}{\text{volume}}$$

The SI unit of density is kg/m^3, but density is often expressed in g/cm^3 (a.k.a. g/mL). The density of a substance varies with temperature. Water is most dense at 4.0°C where ρ equals 1.000 x 10^3 kg/m^3 or 1.000 g/cm^3.

Specific Gravity

Specific Gravity is defined as the density of a substance ρ_x divided by the density of water.

$$\text{specific gravity}_x = \frac{\rho_x}{\rho_{water}}$$

Specific gravity, like density, varies with temperature, but since water's density does not stray far from 1.0 kg/m^3, specific gravity will tend to approximate density. Note that like the radian, specific gravity is a unitless number.

Pressure

Pressure P is defined as the force per unit area.

$$P = \frac{F}{A}$$

The SI unit of pressure is the Pascal *Pa*, which equals a N/m^2. Pressure is also commonly expressed in atmospheres *atm*, which is the pressure, due to the weight of the atmosphere, at sealevel. One atm is approximately equal to 10^5 Pa.

Hydrostatic pressure is the force per unit area created by the weight of a fluid on itself and on its surroundings. The term *columb height h* is often used to refer to the depth of the fluid. Hydrostatic pressure may be found by multipling the density of the fluid, by the acceleration due to gravity, by the columb height.

$$\boxed{P = \rho g h}$$

►Note that as we move downward through a fluid the pressure increases in directly proportion to the depth.

►For a given fluid it is the columb height that determines pressure. The total volume or shape of the fluid will not effect the pressure if the depth of fluid remains constant.

If the pressure at one depth is known P_o, it is possible to find the pressure at another depth P if the vertical displacement Δh between the two positions can be determined:

$$P = P_o + \Delta P \qquad \Delta P = \rho g \Delta h$$

$$\boxed{P = P_o + \rho g \Delta h}$$

A flat tire will have an *absolute* pressure of approximately one atmosphere, but a pressure gauge would read zero atm. That's because the gauge reads the difference between tire pressure and atmospheric pressure P_{atm} ; this is called the *guage pressure*. Now if we increase the tire's pressure by 2 atm, the guage will read 2 atm, but the absolute pressure will be 3 atm. **Absolute pressure** P is the actual pressure. **Gauge pressure** ΔP_g the pressure diference, found by subtracting atmospheric from absolute pressure.

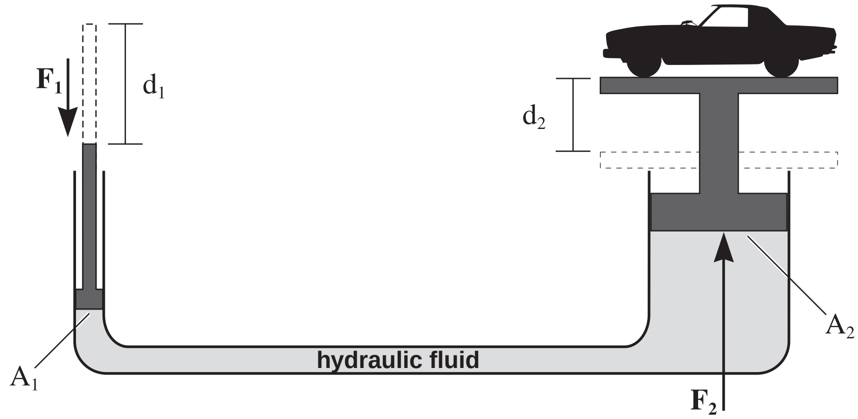
$$\Delta P_g = P - P_{atm}$$

7.2 Properties of Fluids

Pascal's Principle

Pascal's Principle states that *an increase in pressure applied to one portion of an enlcose fluid will cause an equivalent increase in pressure through out the fluid.*

Pascel's principle can be demonstrated in a device called a hydrolic lift. This device makes it possible to elivate a car using a force which is only a fraction of the car's weight. The lift uses an incompressible fluid called hydraulic fluid.



The force F_1 is applied on the left end of the lift to a pistoned of area A_1. This creates an increase in pressure through out the fluid which is equal to F_1/A_1. At the right end of the lift this pressure acts on a larger cylinder with area A_2. Since the $A_2 > A_1$, it follows that $F_2 > F_1$:

$$P_2 = P_1, \qquad \frac{F_2}{A_2} = \frac{F_1}{A_1}, \qquad \frac{F_2}{F_1} = \frac{A_2}{A_1}.$$

►Note that the force generated is directly proportional to the area.

Since the fluid is imcompressable the volume displaced on the left will equal the volume displaced on the right. Recall that area times displacement equals volume:

$$V_2 = V_1, \qquad A_2 d_2 = A_1 d_1, \qquad \frac{A_2}{A_1} = \frac{d_1}{d_2}.$$

►Note that the area is inversly proportional to the displacement.

When using an incompressable fluid the work done on the right will be equal to that done on the left:

$$W_2 = W_1, \qquad F_2 d_2 = F_1 d_1, \qquad \frac{F_2}{F_1} = \frac{d_1}{d_2}.$$

►Note that force is inversly proportional to displacement.

To summarize, the smaller force corresponds to a smaller area and a larger displacement. Or:

$$\boxed{\frac{F_2}{F_1} = \frac{A_2}{A_1} = \frac{d_1}{d_2}}$$

Archimede's Principle

Archimede's Principle states that *a body completely or partially immersed in a fluid will experience an upward force equal to the weight of fluid which it displaces.*

Imagine a 1.0 m^3 cube which is totally submerged in water. Since the bottom face is 1 m below the top face the pressure at the bottom must be greater. Or:

$$(\rho_{H_2O} = 10^3 \text{ kg/m}^3, \text{ let } g = 10 \text{ m/s}^2, \Delta h = 1.0 \text{ m})$$

$$\Delta P = \rho g \Delta h, \qquad \Delta P = (10^3)(10)(1), \qquad \Delta P = 10^4 \text{ Pa}.$$

Since pressure equals force over area:

$$\Delta P = \Delta F/A, \qquad \Delta F = \Delta P \cdot A, \qquad \Delta F = (10^4)(1) = 10{,}000\text{ N}.$$

The cube is subject to an upward force of 10,000 N, this *upward* force is known as the *bouyant force*. According to Archimede's principle the **bouyant force** is equal to the weight of the fluid which is displaced. Solving for the weight of water displaced by the cube should also give us 10,000 N:

$$\text{mass} = V\cdot\rho, \quad \text{weight} = V\cdot\rho\cdot g, \qquad \text{weight} = (1)(10^3)(10) = 10{,}000\text{ N}. \qquad \text{Or:}$$

$$\boxed{\text{buoyant force} = (V_{object})(\rho_{fluid})(g)}$$

Now imagine that the density of this cube is less than the density of water. The weight of the cube, therefore, will be less weight of the water which it displaces.. The net force on the block will be the difference between its weight and the bouyant force. Since the bouyant force is greater the cube will rise to the surface. Once at the surface the cube will continue to rise up to that point where its weight is exactly counterbalanced by the bouyant force. What fraction of the object's volume will remain submerged? Let's set the bouyant force equal to the weight of displaced fluid to find out:

$$(V_{object})(\rho_{fluid})(g) = (V_{submerged})(\rho_{object})(g),$$

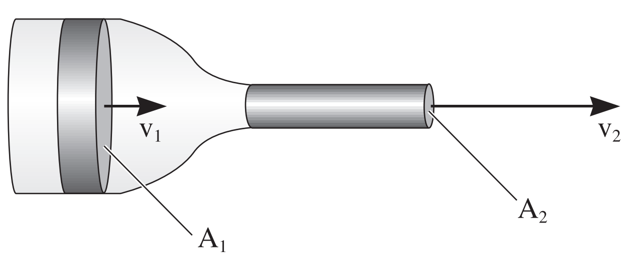
$$\boxed{\frac{V_{submerged}}{V_{object}} = \frac{\rho_{object}}{\rho_{fluid}}}$$

Continuity Equation and Bernoulli's Equation

Continuity Equation

Pascel's principle predicts that $A_1d_1 = A_2d_2$ (volume = area x displacement). For a fluid exibiting a steady, incompressable flow, the same relationship applies, the volume flowing through any cross section of the tube must equal the volume flowing through any other cross section. We derive the **equation of continuity** by dividing $A_1d_1 = A_2d_2$ through by time (note that d/t = velocity):

$$\boxed{A_1v_1 = A_2v_2}$$



The terms A_1v_1 and A_2v_2 represent the **volume flow rate** Q, which is the volume of fluid that moves past a cross section of pipe in a given unit of time. These terms are represented in Figure#■ by two shaded cylindricals of equal volume. Since the tube has a lower cross sectional area on the right, the fluid passing through this constriction will have a larger velocity.

Bernoulli's Equation

Bernoulli's Equation describes the relationship between pressure, velocity, and height in a fluid as it moves through a tube, but it is also applicable to fluids that flow around objects such as air flowing around the wing of a plane. It is valid for smooth flow *(laminar flow)*, with minimum internal resistance *(viscousity)* under incompressable conditions. On the MCAT you may consider all liquids and gas to display incompressable flow, unless you are told otherwise.

$$\ldots \boxed{P + \tfrac{1}{2}\rho v^2 + \rho g h = \text{constant}} \ldots$$

■ Bernoulli's Equation predicts that as the velocity and the height of a fluid increase, its pressure will decrease.

example ►

Viscosity

Viscosity η is a measure of a fluid's resistance to flow as layers of fluid slide past each other. This resistance is largely caused by attractive intermolecular forces *IMFs* which generate internal friction. Gasses, having much lower attractive IMFs than liquids, have small viscosities. Viscosities of liquids cover a broad range, which approximate the strenght of the liquid's IMFs. The SI unit of viscosity is $N \cdot s/m^2$.

Pouseille's equation can be used to determine the viscosity of a fluid flowing through a tube of radius *r*and length *l*. The pressure differential between each end of the tube is given by *ΔP*. *Q*, the volume flow rate, is measured in m^3/s in the SI system.

$$\eta = \frac{\Delta P \pi r^4}{Q l}$$

Laminar and Turbulent Flow

Laminar flow occurs when a fluid moves smoothly without the formation of whirlpools. If vortices do occor the flow is called **turbulent.** The **Reynolds number** χ is used to predict the type of flow. Laminar flow is expected when $\chi < 0$, and turbulent flow when $\chi > 2000$.

$$\chi = \frac{2rv\rho}{\eta}$$

Note that turbulence is expected to increase, when all variable *except viscosity* increase. In addition, turbulance is more likely when a fluid encounters abrupt obsticals or sharp changes in te the direction of its path.

Surface Tension

Attractive IMFs cause the surface of fluids to behave different from the bulk fluid. Molecules situated in the bulk of the fluid experience no net force since they are uniformly surrounded by other molecules. Molecules at the surface, however, experience a net force directed back into the bulk of the fluid. The measure of this force is the **surface tension.** Surface tension causes fluids to behave as if they had a skin (like old pudding), making it possible to float a pin on water. Surface tension also causes fluids to wrap their surface around the bulk of the fluid, resulting in the convex shape of a droplet of water on a newly waxed car. In the absence of all external forces, all fluids will become spherical (in space for example) in order to minimize their surface area.

7.3 Elastic Properties of Solids

A substance will maintain its shape when forces external to it are counterbalanced by internal forces. These internal forces may be conceptualized as miniture springs linking together adjacent atoms within a substance.

When external forces exceeds internal ones, objects begin to deform. If force is applied slowly, and does not exceed a critical limit, the substance will return back to its original shape once the force is removed.

Young's Modulus

The degree to which a substance changes shape in response to an applied force is quanified by a number called a **modulus. Young's modulus** *Y* is used when a force is applied in one dimension perpendicular to the surface of a substance. Youngs modulus is defined as stress over strain; where stress is the force per unit area (F/A), and strain is the fractional change in length($\Delta L/L$)..

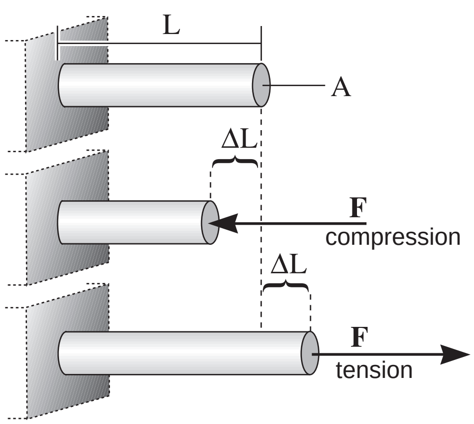


Figure 7.1 Equal and opposite forces are present on both sides of the rod. To simplify this example, only the forces on the right side are shown.

$$Y = \frac{\text{stress}}{\text{strain}} \qquad \text{stress} = F/A \qquad \text{stain} = \frac{\Delta L}{L}$$

$$Y = \frac{F/A}{\Delta L/L}$$

Equation 7.1

A given material has two values for Young's modulus, depending on the type of force which is applied. Compressive forces cause L to decrease, while tensile forces (tension) cause L to increase.

►Note that the change in length ΔL, is directly proportional to L, and inversly proportional to A and Y.

Shear Modulus

Imagine pushing on the spine of a book so that the top of the spine moves forward while the bottom remains in place. The change in the book's shape is characterized as shearing deformation. The **shear modulus** *S* is defined below:

$$S = \frac{F/A}{\Delta X/L}$$

Equation 7.2

F/A is the stress, and $\Delta X/L$ is the strain. *ΔX* represents the displacement, and *L* is the object's thickness.

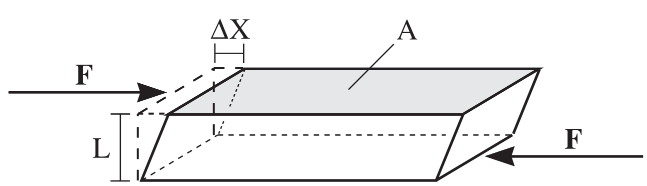


Figure 7.1

Bulk Modulus

When an object is emersed in a fluid it experiences a uniform increase in the force per unit area across its entire surface. This stess, commonly known as **pressure** *P*, will cause the object to contract, thus resulting in a decreased volume. The **bulk modulus** *B* is a measure of this change.

$$\boxed{B = -\frac{\Delta P}{\Delta V/V}}$$

ΔP represents the increase in pressure, and $\Delta V/V$ the fractional decrease in volume. In order to make B a positive number, a negative sign is used in Equation#■.

product

Temperature and Heat

Temperture T is a quantitative measure of atomic and molecular motion. When an object at a higher temperature is placed in contact with an object of lower temperature, energy will be transfererd to the cooler body; this flow of energy is known as **heat** Q. Heat is a form of energy which is store through out the molecular, atomic, and nuclear structure of a substance. The flow of heat underconstant pressure is known as the **enthalpy change** ΔH, in chemistry texts.

8.1 Temperature

Temperature Scales

The **Kelvin** K is a base unit for temperature measurement in the SI system. The Kelvin system has no negative temperatures since zero Kelvin begins at absolute zero — at temperature which can be approached but never reached. In all calculations where T rather than ΔT is required the Kelvin scale *must* be used.

In the Celsius (a.k.a. Centigrade) *°C* scale, the normal freezing point and boiling point of water are defined at 0°C and 100°C respectively. The magnetude of a temperature change in the Celsius scale is the same as the Kelvin scale i.e., a *temperature change* of 100 K is equal to a temperature change of 100°C, but 100 K is not the same *temperature* as 100°C. In calculations where ΔT is required the Celsius or Kelvin scales may be used.

The Fahrenheit *°F* scale although no longer used by most of the world is still in use in the United States. A change of one degree in the Fahrenheit scale is equal to a change of 5/9 of a degree in the Celcius or Kelvin scale.

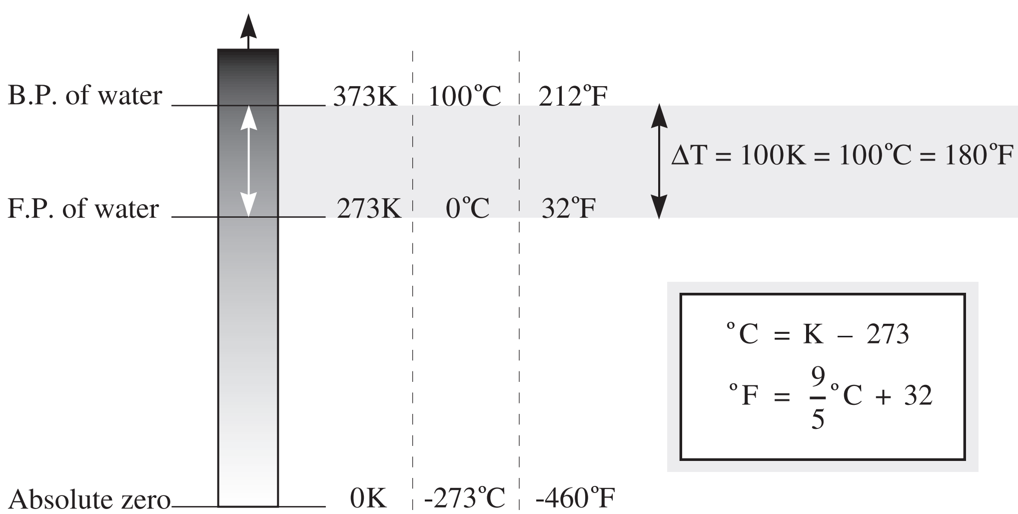


Figure 8.1 Key points on the three temperature scales and the equations which allow conversion from one scale to another.

$$\boxed{K = {}^\circ C + 273}$$

Equation 8.1 Conversion to Kelvin should always be made when a quantitative comparison between two or more temperatures is made.

Thermal Expansion

Most substances expand when they are heated, and contract when cooled. The degree to which they do so is quantified by the **coefficinet of linear expansion** *α,* and the **coefficient of volume expansion** *ß*. The change in a material's length *ΔL,* may be found if its original length *L* is known along with α and ΔT.

$$\boxed{\Delta L = \alpha L \Delta T}$$

The change in a material's area *ΔA* may be found by:

$$\Delta A = (2\alpha) A \Delta T.$$

The change in a materials volume *ΔV* may be found by:

$$\Delta V = (3\alpha) V \Delta T.$$

Since 3α is equal to ß we get:

$$\boxed{\Delta V = \beta V \Delta T}$$

The units for α and ß may be expressed in $°C^{-1}$ or K^{-1}. Since these represent changes in temperature, they are equivalent units.

8.2 Heat

Since heat is a form of energy the units used to descibe energy and heat are the same. While the SI unit is the **Joule** *J*, the **calorie** *cal* is also in used. When used in the context of nutrition the *"food" Calorie* is capitalized to distiguish it from the calorie. A *food Calorie* is equal to one kilocalorie *kcal,* which is 1000 cal. The relationship between Joules and calories is:

$$\boxed{4.18\ J \approx 1.00\ cal}$$

Specific Heat a.k.a. specific heat capacity

Specific heat *c* is a measure of the quanity of heat required to raise the temperature of one gram of a substance by one degree (°C or K). The SI unit of specific heat is J/kg°C, but the units of cal/g°C are in common in the field of chemistry. The heat flow *Q* into a material of mass *m* can be found through the use of Equation#■ :

$$\boxed{Q = mc\Delta T}$$

►The specific heat of a substance depends on its phase i.e., the specific heat of H_2O (g) ≠ H_2O (l) ≠ H_2O (g).

The calorie is defined as the quantity of heat required to raise the temperature of 1g of water from 14.5°C to 15.5°C. ►Since the specific heat of water does not change much with temperature, <u>*the specific heat of water is 1.00 cal/g°C or 4.18 J/g°C.*</u>

assignment ►

Heat Capacity

Heat capacity *C* is the quantity of heat required to raise the temperature of a substance by one degree. The SI unit of heat capacity is J/°C. Unlike specfic heat, heat capacity depends on the amount of matter present. When the mass of a substance is known, its heat capacity may be obtained by multpling its mass by its specific heat,

$$C = mc.$$

Substituting into Equation#■ gives:

$$\boxed{Q = C\Delta T}$$

assignment ►

Latent Heat (a.k.a. heat or enthapy *H)*

The addition or removal of heat from a substance does not always result in a temperature change, instead the flow of heat may result in the reorganization of IMFs. This is known as a **phase change.**

The quaninty of heat that must be added or removed when 1 kg of a substance converts from one phase into another is known as **latent heat** *L*. The SI unit of latent heat is J/kg. The terms *fusion, vaporization,* and *sublimation* are are appended on to the term *latent heat,* to refer to: *(s)* $\rightleftharpoons$ *(l),* or *(l)*$\rightleftharpoons$*(g),* or *(s)*$\rightleftharpoons$*(g)* respectively*;* where *(s)* = the solid phase*, (l)* = the liquid phase*, (g)* = the gas phase*,* and "$\rightleftharpoons$" represents a change of phase in either direction.(See Section#■ for additional information).

assignment ►

Heat Transfer

Heat transfer occurs by three mechanisms: conduction, convection, and radiation. If one end of a metal rod is heated, the other end with will also heat up. This mechanism of heat transfer is called **conduction.** In conduction there is no net movement of material, instead energy transfer occurs through the *collisions of adjacent atoms.* The measaure of a material's ability to conduct heat at a given temperature is given by a constant known as the **thermal conductivity** *K***.** Most metals are good conductors and have high values for *K,* while thermal insulators like asbestos or air have low thermal conductivities.

When soup is heated on a stove, the hotter soup near the bottom rises, while the colder soup near the top sinks. This form of heat transfer is called **convection.** Since convection involves *the flow of a material* it is only observed in fluids (liquids and solids). The process of convection is more rapid than that of conduction.

When you turn your face to the sun you can feel its warmth. This example of heat transfer takes place through a process called **radiation**. Radiation is the most rapid form of heat transfer because energy is being transmitted via *electromagnetic waves*. No contact of atoms, flow of material, or external medium is required for radiative energy transfer to take place.