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Translational Motion

Thanks for downloading THE MCAT EXCELERATOR™. Although they are only rough drafts, I hope these chapters will help you out on the MCAT. I'd love to hear any comments, corrections or suggestions you may have.
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■ The chapters on physics which follow, require some quantitaive skills that were introduced in the preceding chapter.

■ In the text below, *s* is used to represent distance, and *d* is used represent displacement. In college physics texts, the use of these symbols has not been standardized.

1.1 Basic Concepts

Speed

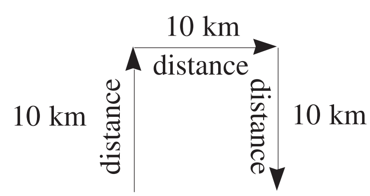
■ **Average speed** is a scalar quantity. It is defined as distance, *s,* over the change in time, Δt.

$$\text{Average speed} = \frac{s}{\Delta t}$$

assignment ►

A car travels 10 km north for 30 minutes, than 10 km east for 15 minutes, then 10 km south for 15 minutes. What is the average speed of the car over the entire trip in km/hr?

solution:

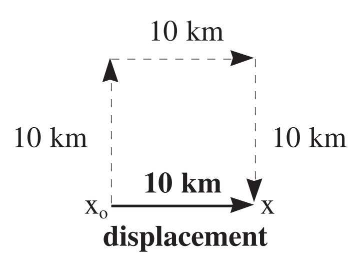


The total distance traveled is 30 km. Dividing by the total elapsed time of 1.0 hr, gives an average speed of 30 km/hr.◆

Displacement

■ **Displacement**, *d*, is a vector quantity that represents change in position. It is graphically depicted by an arrow with its tail on the initial position, x_o, and its head on the final position, *x*.

$$d = x - x_o$$



Velocity

■ **Average velocity**, $\bar{v}$, is a vector quantity. It is defined as the displacement, over the change in time.

$$\bar{v} = \frac{d}{\Delta t}$$

assignment ►

For the previous example what is the average velocity over the course of the trip in km/hr?

solution:

The displacement is 10 km due east. Dividing by the total elapsed time of 1.0 hr, gives, an average velocity of 10 km/hr due east. ◆

Instantaneous Velocity

■ **Instantaneous velocity** is a vector quantity. As Δt approaches zero, the average velocity becomes the instananeous velocity.

$$v = \lim_{\Delta t \to 0} \frac{d}{\Delta t}$$

When an objects position is plotted against time, the slope of the curve at any given instant in time, is the instantaneous velocity.

Acceleration

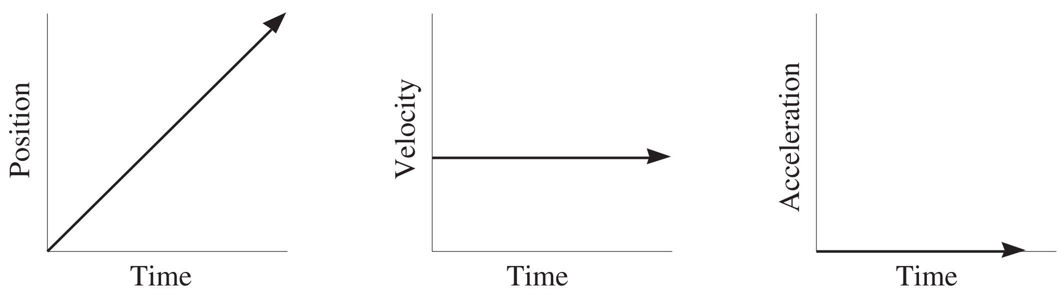
■ **Acceleration**, *a*, is a vector quantity. It is defined as the change in velocity, over the change in time. A change in either the magnitude or the direction of the velocity is an acceleration.

$$a = \frac{\Delta v}{\Delta t}$$

Uniform acceleration occurs when the change in velocity with respect to time is a constant.

Zero Acceleration

When the velocity remains constant there is no acceleration. The graphs below depict this situation.



Note that the slope of the graph on the left is equal to the velocity. Since this is a linear plot, the velocity remains constant over time, as depicted in the central graph. Since the velocity is constant, the acceleration is zero.

■ When there is no acceleration the displacement is equal to the elapsed time multiplied by the velocity.

$d = \Delta t \cdot v$

Motion in One Dimension

When the velocity and the acceleration have the same or opposite orentations, the direction of motion will not deviate from a straight line. The following equations apply to straight line motion with uniform acceleration.

Each of the following equations contains four of the five variables listed below.

d = displacement **v_o** = initial velocity **v** = final velocity **a** = acceleration

t = elapsed time.

The variable which is not present in an equation appears in parenthesis.

1. **(d)** $v = v_o + at$

2. **(v_o)** $d = vt - ½at^2$

3. **(v)** $d = v_ot + ½at^2$

4. **(a)** $d = ½(v_o + v)t$

5. **(t)** $v^2 = v_o^2 + 2ad$

A useful convention is to assign any vector quantity that is directed upward or to the right a positive value, and assign a negative value to any vector oriented downward or to the left.

Free fall occurs when an object moving close to the earth's surface experiences an acceleration due to gravity. No other forms of acceleration, such as the effects of air resistance, are considered.

question ►

A ball is thrown directly upward with a velocity of 20 m/s. What is the maximum height the ball will reach above the point from where it was released? The acceleration due to gravity may be approximated as $10\ m/s^2$.

solution:

We will assign v_o a positive value since it is directed upwards, and the acceleration due to gravity, a negative value since it is always directed downwards.

Since the direction of the acceleration opposes the velocity, the ball's velocity will decrease in magnitude until the ball reaches its maximum height. At this point, when upward motion stops, v will equal zero.

d =	v_o =	v =	a =	t =
?	+20 m/s	0	$-10\ m/s^2$	~

Since we are not interested in t, we use equation 5.

$$0^2 = (20)^2 + 2(-10)d$$

$$-400 = -20d$$

$$d = +20\ m$$ ◆

question ►

Using the original information from the previous problem, find the time required for the ball to reach its maximum height.

solution:

d =	v_o =	v =	a =	t =
~	+20 m/s	0	$-10\ m/s^2$	?

Since we are not interested in d, we will use equation 1.

$$0 = 20 + (-10)t$$

$$-20 = -10t$$

$$t = 2.0\ s$$ ◆

The information contained in the preceding problems is graphically depicted below.

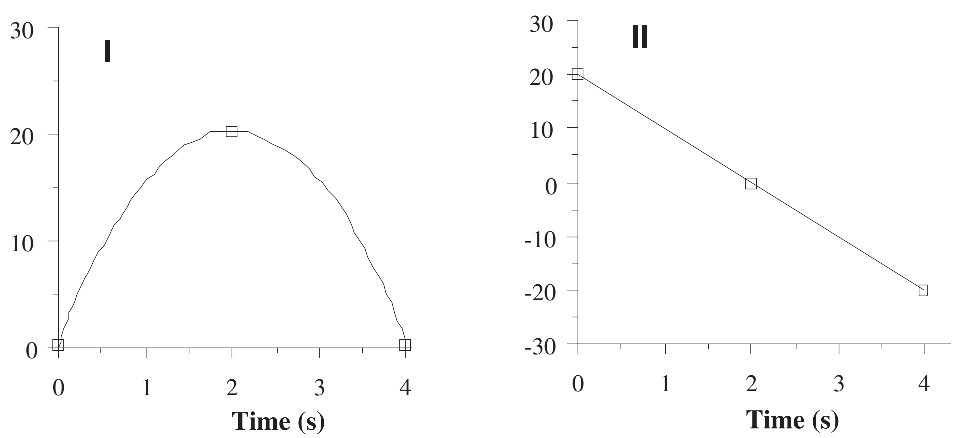
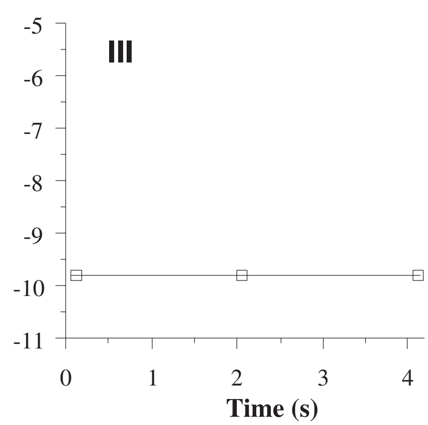


Figure 1.1

In graph I of Figure #, notice that the maximum height is reached at 2.0 seconds. At this point the ball will stop, and then and reverse direction. The slope, and thus the velocity, at t = 2.0 s is zero. This is depicted in graph II. Notice that at t = 2 s the velocity is zero. After 2.0 s, the velocity will be negative; this indicates motion in the downward direction.

The linear plot in graph II, indicates a constant acceleration, since the change in velocity with respect to time (slope) is the acceleration. This is depicted in graph III. The acceleration is negative because it is directed downwards.



Projectile Motion

The ball in the previous example had an initial velocity that was directed vertically upward. Since the acceleration due to gravity also acts vertically, the ball's velocity remained vertical. When the velocity and the acceleration do not both act along the same line, however, a curved, rather than a linear trajactary, result. **Projectile motion** occurs when an object follows a curved path during free fall

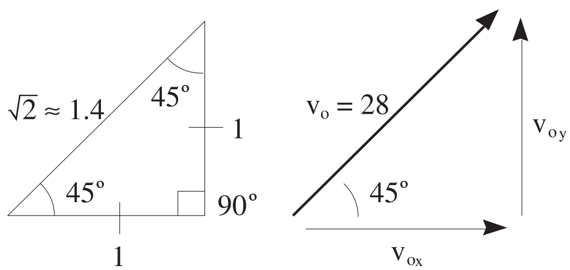
question ►

A ball is thrown, from ground level, with a velocity of 28 m/s, at an angle 45° above the horizontal. How far from the point at which the ball was thrown will it land?

solution:

The distance the ball travels in the horizontal direction is called the *range*. In order to solve for the range, it is first necessary to resolve the initial velocity into its x and y components.

The initial velocity may be represented as the hypotenuse of an iscosceles right triangle, with a magnetude of 28 m/s. Since the two legs of this triangle are equal, the initial velocity in the vertical direction, v_{oy}, will be equal to the initial velocity in the horizontal direction, v_{ox}. The value of v_{oy} and v_{ox} may be found by the use of the sin and cos functions respectively.. Sin 45° and cos 45° both equal $\sqrt{2}/2$., which is approximately 0.7. By multiplying 28 m/s by this factor, the x and y components of the intial velocity are both found to both be to 20 m/s.



An isosceles right triangle

$$\sin 45^\circ = \frac{v_{oy}}{28}$$

$$\cos 45^\circ = \frac{v_{ox}}{28}$$

$$v_{ox} = v_{oy} = 28\left(\frac{\sqrt{2}}{2}\right) = 28(\sim 0.7) \approx 20$$

An alternative method for obtaining the x and y components of the velocity is shown below.

$$v_{ox} = v_{oy} = 28 \text{ x } \frac{1}{\sim 1.4} \approx 20$$

In a right iscoceles triangle, the length of the legs as compared to the length of the hypotenuse is 1.0 : ~1.4. Therefore the length of the legs may be obtained by dividing 28 m/s by 1.4.

In projectile motion the x and y components of the velocity act independently. We can thus use v_{oy}, to find the time the ball remains in the air, and then use v_{ox} to find the range.

In the previous question the vertical velocity was also 20 m/s. Refering back to graph I, we found it took 2.0 s for the ball to reach its maximum height. Another 2.0 s is required for the ball to return to ground level. The ball's time of flight, therefore, is 4.0 s. *Notice that the ball's horizontal velocity has no bearing on the length of time the ball remains in the air.*

Since there is no acceleration in the horizontal direction, v_{ox} will remain constant, and the horizontal displacement will equal $\Delta t \cdot v_o$.

$$4.0\,\text{s} \times \frac{20\text{ m}}{\text{s}} = 80\text{ m}$$ ◆

question ►

What will be the instantaneous velocity one second after the ball in the previous qyestion is thrown?

A. $\sqrt{200}$ m/s

B. $\sqrt{300}$ m/s

C. $\sqrt{400}$ m/s

D. $\sqrt{500}$ m/s

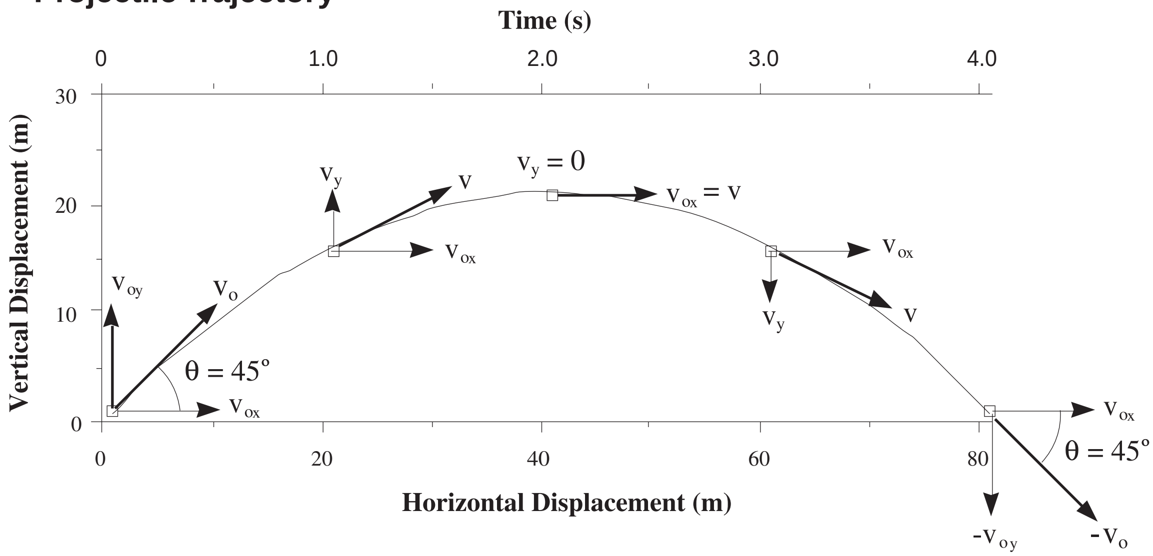
solution:

To find the instantaneous velocity we will need to know v_x and v_y at t = 1.0 s. Since v_x = 20 m/s at all times, v_x at t = 1.0 s is 20 m/s. Since the acceleration is a uniform -10 m/s,^{2} the change in velocity during any one second interval will be -10 m/s. Since v_{oy} is 20 m/s ,v_y at t = 1.0 s is 10 m/s,

V_x and v_y make up the legs of a right triangle, with v as the hypotenuse. Using the Pythagorean Theruom we get $v^2 = v_x^2 + v_y^2$ or $v^2 = (20)^2 + (10)^2$. Solving for v gives answer **D**, $\sqrt{500}$ m/s.◆

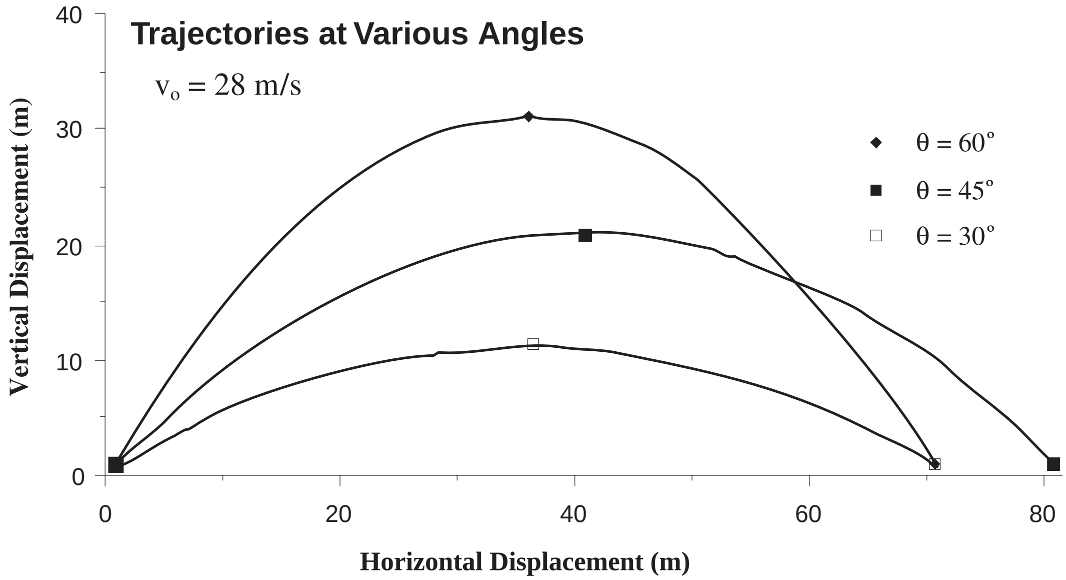
The path followed by the ball in the previous problem is diagramed below. Note that v_x remains constant since the horizontal acceleration is zero. During the first two seconds v_y decelerates since the acceleration due to gravity opposes this component of the velocity. At t = 2.0 the ball stops moving in the vertical direction, but continues forward with constant horizontally velocity. It is at this point that the ball is at its maximum height. For the final two seconds the ball velocity increases in the downward direction. At t = 4.0 seconds the ball sticks the ground. At this point v_x is still 20 m/s, but v_y is has reversed direction and is now -20 m/s.

Projectile Trajectory



The instantaneous velocity, *v,*at any given point in time is the vector sum of v_x and v_y. Note that the magnitude of the final velocity is equal to the magnitude of the initial velocity. The direction has changed however. The initial velocity was directed 45° *above* the horizontal, at t = 4.0 s, the final velocity is directed 45° *below* the horizontal.

■ The range will be greatest when $\theta = 45°$. For pairs of angles that satisfy the equation $\theta = 45° \pm x°$, the range will equal. For example a 60° and a 30° angle will result in the same range, because they both differ from a 45° angle by 15°. The diagram and table below refer to 30°, 45° and 60°.



Angle	Range	Time in Flight	Maximum Height
30°	~70 m	~ 3 s	~ 10 m
45°	~ 80 m	~ 4 s	~ 20 m
60°	~ 70 m	~ 5 s	~ 30 m

Force and Newton's Laws

2.1 Newton's Three Laws

Force

■ **Force** is a vector quantity. When an object is pushed or pulled it is subject to a force. Forces may act through direct contact, such as when ball is kicked, or indirectly as when an object experiences a downward pull due to gravity. The unit of force is the Newton, *N*, and is equivalent to a kg·m/s^2.

When two or more forces act upon an object, the resultant force is obtained by vector addition (←Section #) This resultant force is known as the **net force**, ΣF. For example in the accompaning diagram the net force is determined by vector addition of force x and y.

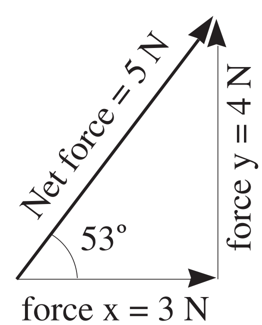


Figure 2.1

Newton's First Law of Motion

■ **Newton's first law** states that an object at rest tends to stay at rest, and an object in motion tends to stay in motion, unless acted on by a net force.

Newton's first law may be restated as, "When the net force is zero, the acceleration is zero".

Mass and Inertia

■ **Inertia** is the tendency of an object to resist a change in velocity.

■ **Mass**, a scalar quantity, is a quantitative measurement of an object's inertia.

Newton's Second Law of Motion

■ **Newton's second law** states that when an object experiences a net force, it will accelerate in the same direction as this force, and the magnitude of its acceleration ,will be directly proportional to its mass.

$$\boxed{\Sigma F = ma}$$

Equation 2.1

question ►

A 1 kg object experiences a 10 N force to the right, and a 6 N force to the left. What will be the acceleration of this object?

solution:

The net force is 6 N - 4 N = 2 N. Substituting into $\sum F = ma$, gives, 2 N = 1 kg·a. Therefore, the acceleration, *a*, = 2 m/s^2, directed to the right.◆

Newton's Third Law of Motion

■ When one body exerts a force on a second body, the second body excerts an equal force, in the opposite direction upon the first body.

One example which illustrates the third law, is that of a swimmer, doing laps in a pool. As she pushes off the wall, the wall pushes back with an equal, but opposite force. This force causes the swimmer to experience an acceleration directed away from the wall, thus allowing her to reverse direction and pick up speed.

2.2 Gravitational Force

All particles exert equal and opposite attractive forces on each other. The magnitude of the force acting between two particles of masses m_1 and m_2, separated by a distance *r*, is given by **Newton's law of universal gravitation**:

$$F = G\frac{m_1 m_2}{r^2}$$

Equation 2.2

G is the universal gravitational constant. The value of G has been determined experimentally to be 6.67×10^{-11} N·m^2/kg^2.

Acceleration due to Gravity

question ►

In terms of *G;* the mass of the earth, *M;* and the radius of the earth, *R;* solve for the acceleration due to gravity on an object resting on earth's surface.

solution:

Let m = the mass of the object. Setting the force in the Newton's second law, equal to the force in the law of universal gravitation gives:

$$ma = G\frac{M\,m}{R^2}.$$

Canceling out the mass of the object yields:

$$a = G\frac{M}{R^2}$$ ◆

■ The acceleration on, or near, the earth's surface is denoted by the symbol, *g*. The value for g is 9.8 m/s^2. *The value of g is typically given as a positive value in equations, but the downward direction of g is understood.*

$$g = G\frac{M}{R^2}$$

■ Note that the value for g depends only on the radius of the earth, and the mass of the earth.

Weight

The gravitational force exerted on an object by the earth is called its weight, *W*. This force is always directed toward the center of the earth. Referring to the equations above, the weight of an object may be given as:

$$\boxed{W = mg = G\frac{M\,m}{R^2}}.$$

Equation 2.3

question ►

By how much would the weight of an object on the surface of a planet which has twice the mass of earth and half earth's radius, differ from its weight on earth's surface?

solution:

From the equations above we see that weight is proportional to the mass of the planet times the mass of the object, divide by the radius of the planet squared.

$$W \propto \frac{M\,m}{R^2}$$

Since the mass of the object will not change we get:

$$W \propto \frac{M}{R^2}$$

If we let M' be the mass of the planet, and R' its radius we can derive the proportional change (←) in weight for the object on the planet as compared to earth, W'/W.

$$\frac{W'}{W} = \frac{M'/M}{(R'/R)^2}$$

Since M'/M = 2, and R'/R = 1/2, we get:

$$\frac{W'}{W} = \frac{2/1}{(1/2)^2} = 8$$

The object's weight would be eight times greater on the planet, than on earth.◆

Apparent Weight ↓ don't need, convert to passage

Your apparent weight, W_a, is the value an accurate bathroom scale would read while you stood on it. If you stand on the scale in an elevator which is at rest, or is moving at constant velocity, W_a will equal W. If, however, the elevator is accelerating upward, then $W_a > W$. On the other hand if the elevator is accelerating downward, $W_a < W$.

Your apparent weight may be calculated by multiplying your mass by the sum of the magnitude of g plus the acceleration of the elevaton.

$$\boxed{W_a = m[g + \vec{a}]}$$

In the above equation the values of W_a and g are always positive. Their directions are understood to be downward. The acceleration, a, is positive if upward, and negative if downward.

question ►

A 50 kg woman stands on an accurate bathroom scale on an elevator moving upward at 20 m/s. After 2.0 seconds the the elevator has an upward velocity of 10 m/s. Assuming uniform acceleration, and a value of g equal to 10 m/s^2, what is the woman's apparent weight?

solution:

The average acceleration is, $(v - v_o)/t$, which is (10 m/s - 20 m/s)/2 s = -5 m/s^2. (Note that the acceleration is directed downward, because the upward velocity is decreasing)

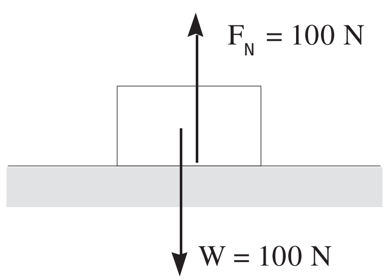
$$W_a = 50\ kg\ [10\ m/s^2 + (-5\ m/s^2)]$$
$$W_a = 250\ N$$

◆

2.3 Friction

The Normal Force

When a 100 N object rests on a table, Newton's third law predicts that the downward force of the weight must be opposed by an equal and opposite force coming from the tabletop. This force which is directed perpendicularly outward from a surface is called the **normal force**, F_N.



Static Friction

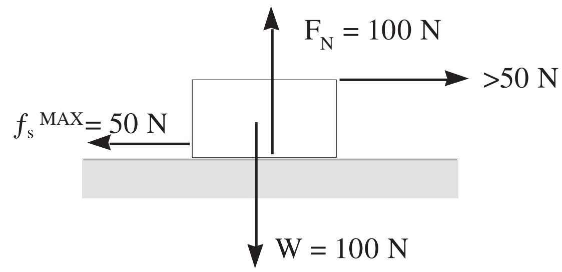
When a horizontal force of 20 N is applied to this object, an equal and opposite horizontal force opposes the applied force. This opposing force is called the **force of static friction,** f_s. The maxium value of f_s is f_s^{MAX} . The value of f_s^{MAX} is a function of F_N and the properties of the two materials which are in contact. The coeffiecent of static friction, μ_s, is a measure of the degree to which the two materials which are in contact will resist the tendency to slide past one another.

$$\boxed{f_s^{MAX} = \mu_s \cdot F_N}$$

Equation 2.4

If the value of μ_s is 0.5. The maximum value of static friction will be, (0.5)(100 N) = 50 N. If the applied force increases from 20 N up to 50 N, f_s will increase correspondingly. When the applied force reaches 50 N, f_s will equal f_s^{MAX}. At this time any further increase of the applied force will cause the object to begin to slide.

Sliding begins when horizontal forces are unequal.



Kinetic Friction

When sliding begins the **force of kinetic friction,** f_k, will oppose the applied force. If the applied force is made equal in magnitude to f_k, then there will be no net force, and the object will slide with a constant velocity. If the applied force is greater than f_k, then there will be a net force acting in the direction of the velocity. This will cause the object to accelerate in the direction of the applied force.

The force of kinetic friction is calculated in much the same way as f_s^{MAX}. The coefficent of kinetic friction is μ_k. The value of μ_k indicates the resistance that will be encountered when sliding one material past another.

$$f_k = \mu_k \cdot F_N$$

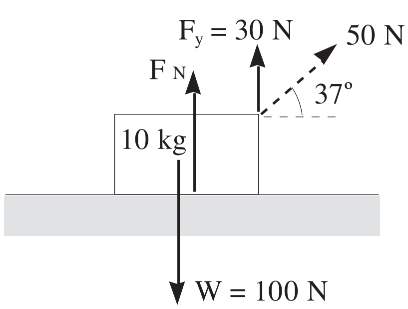
■ For a given pair of materials $\mu_s > \mu_k$. Therefore, the maximum value of static friction will always be larger than the force of kinetic friction. The force required to cause an object to *begin* sliding, is always more than the force required to cause the object to *continue* to slide at a constant velocity.

question ►

Calculate the acceleration of a 10 kg block, which experiences a 50 N force directed 37° above the horizontal. Let g = 10 m/s^2, and $\mu_k = 0.2$.

solution:

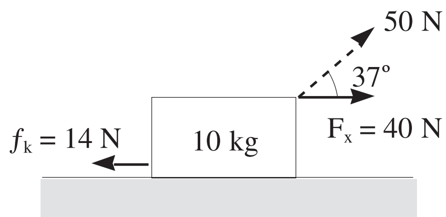
The first step is to create a diagram with all nonhorizontal forces resolved into their y components. In the diagram below the weight is found by mulipling the mass of 10 kg by the acceleration due to gravity of 10 m/s^2. The y component of the applied force, F_y, is found using, sin 37° = F_y/50 N; F_y = 30 N.



$\Sigma F_y = 0$
$F_N + 30\text{ N} - 100\text{ N} = 0$
$F_N = 70\text{ N}$

Since there is no acceleration in the vertical direction, the net force, ΣF_y, is equal to zero. The net force will equal the normal force, plus the applied vertical force, minus the weight. The weight is given a negative sign because it is dirceted downward. The value of the normal force is found to be 70 N.

The next step is to create a diagram with all nonvertical forces resolved into their x components. In the diagram below the force of kinetic friction is found by mulipling the coefficemt of kinetic friction by the normal force to give, (0.2)(70 N) = 14 N. The x component of the applied force, F_x, is found using, cos 37° = F_x/50 N; F_x = 40 N.

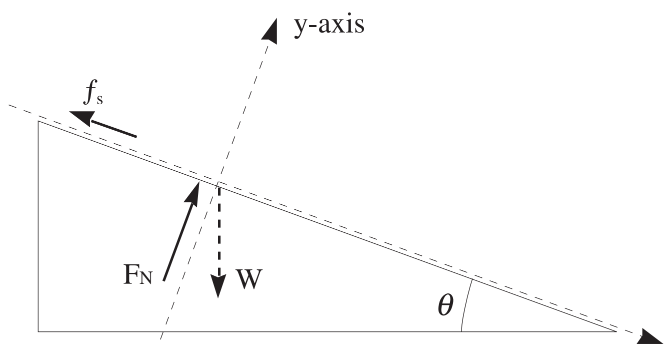


$\sum F_x = ma$
$40\text{ N} - 14\text{ N} = 10\text{ kg}\cdot a$
$a = 3.8\text{ m/s}^2$

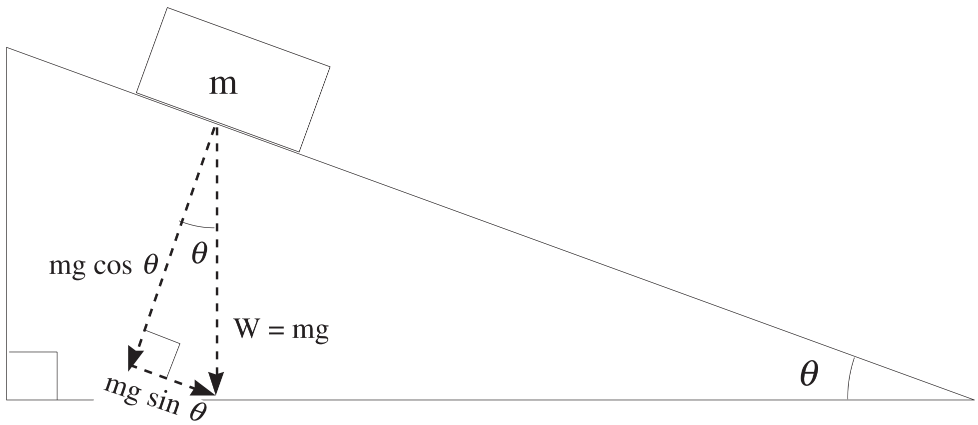
Since we expect an acceleration in the horizontal direction, the net force in the x direction is set equal to the mass times the acceleration. The net force will be equal to the applied horizontal force minus the force of kinetic friction. The value of the frictional force is given a negative value because it is directed to the left. Using 10 kg for the mass gives an acceleration of 3.8 m/s^2.◆

Inclined Planes

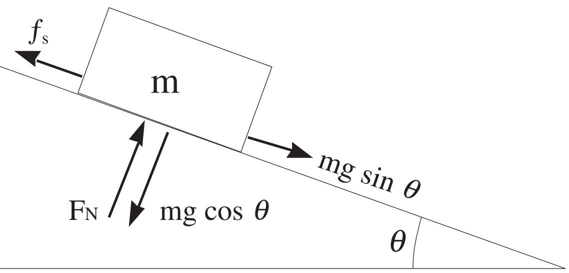
When an object rests on an inclined plane, its weight is directed downward, the normal force is directed perpendicularly outward from the surface of the incline, and the frictional force is directed parrellel to the incline in the upward direction. The x and y axes are respectively oriented pareralle and perpendicular to the surface of the incline.



The weight, which is equal to *mg,* can be resolved into its components on the x and y axis as defined above. The weight is illustrated below as the hypotenuse of a right triangle which is similar to the triangle formed by the incline plane. The x component of the weight is given by mg sin θ, and the y component of the weight is given by mg cos θ.



When the x and y components of the weight are superimposed on the inital diagram, the following relationship of forces is obtained.



Since the block is at rest, the net force in both the x and y directions must be equal to zero.

$\Sigma F_y = 0$, $\quad F_N - mg\cos\theta = 0$, $\quad F_N = mg\cos\theta$.

$\Sigma F_x = 0$, $\quad mg\sin\theta - f_s = 0$, $\quad f_s = mg\sin\theta$.

question ►

What is the minimum value of μ_s required to prevent an object from slipping on an incline plane with an angle of θ?

solution:

To prevent slipping f_s^{MAX} must be equal in magnitude to mg sin θ:

$mg\sin\theta - f_s^{MAX} = 0$, since $f_s^{MAX} = \mu_s \cdot F_N$:

$mg\sin\theta - \mu_s \cdot F_N = 0$, since $F_N = mg\cos\theta$:

$mg\sin\theta - \mu_s \cdot mg\cos\theta = 0$, solving for μ_s gives:

$\mu_s = \sin\theta/\cos\theta$, since $\sin\theta/\cos\theta = \tan\theta$:

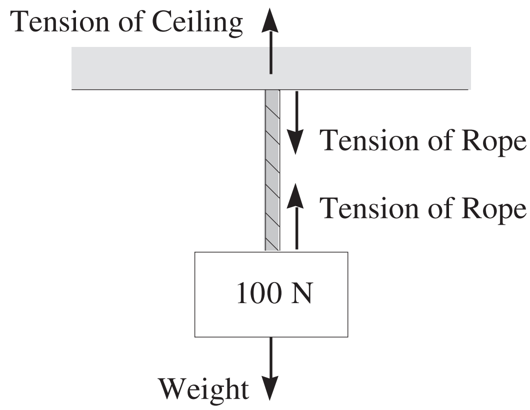
$\mu_s = \tan\theta$.◆

2.4 Tension and Pulleys

Tension

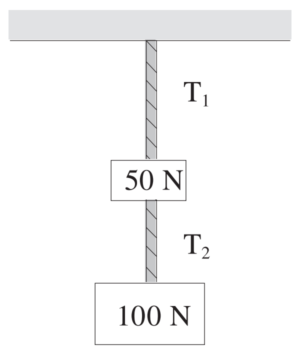
Tension, *T*, is a pulling force. When a length of rope having negligable mass is under tension, both ends of the rope pull with equal force in opposite directions.

In the adjacent example a 100 N object is suspended from a ceiling by a rope. The object's weight of 100 N is directed downward. The rope pulls with a 100 N force, upward on the object and downward on the ceiling. The ceiling resists the downward pull of the rope with a force of 100 N directed upward.

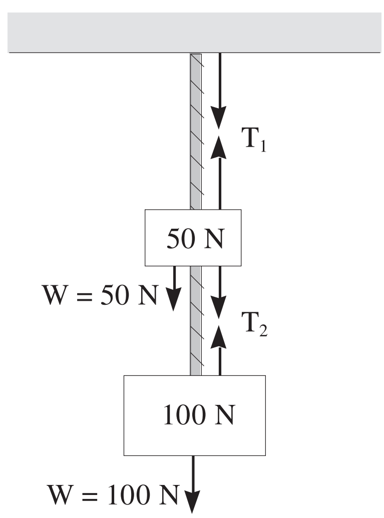


question ►

What are T_1 and T_2 in the diagram below? All objects are at rest.



solution:



Since the 100 N object is at rest, the net force on it must be zero:

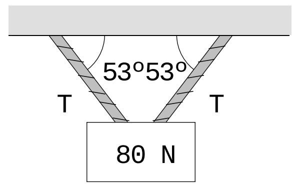
$\Sigma F_{100\text{ N}} = 0$, $T_2 - 100\text{ N} = 0$, $T_2 = 100\text{ N}$.

Since the 50 N object is at rest, the net force on it must be zero:

$\Sigma F_{50\ N} = 0$, $T_1 - T_2 - 50\ N = 0$, $T_1 = 150\ N$.◆

question ►

What is the value of T below? All objects are at rest.

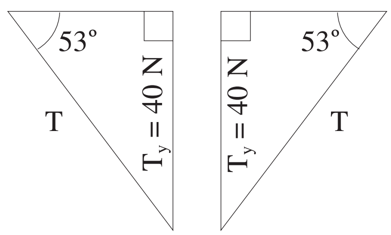


solution:

Since the 80 N object is at rest, the net forec on it must be zero. In the vertical direction the force of the weight will be opposed by the vertical component of the tension, T_y. Each rope has the same value of T_y, since each rope is at the same angle.

$\Sigma F_y = 0$, $T_y + T_y - 80\ N = 0$, $T_y = 40\ N$.

The total tension, *T*, is obtained by applying the sine function as indicated below.



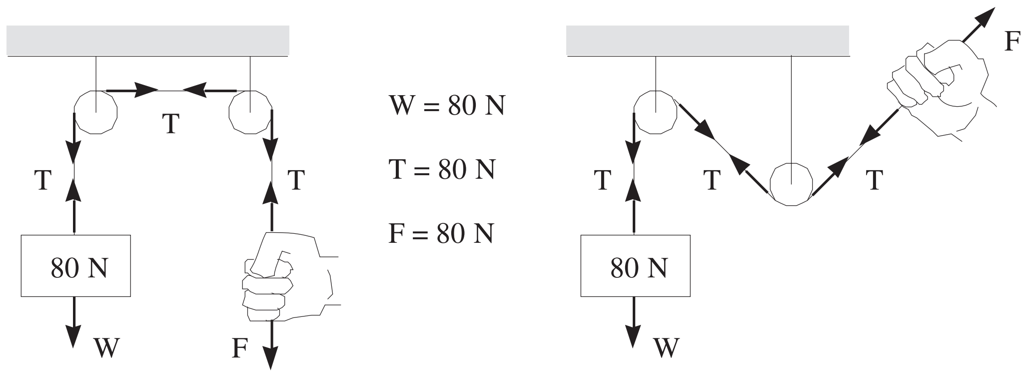
$$\sin 53° = \frac{40\ N}{T}$$

$$T = 50\ N$$ ◆

Pulleys

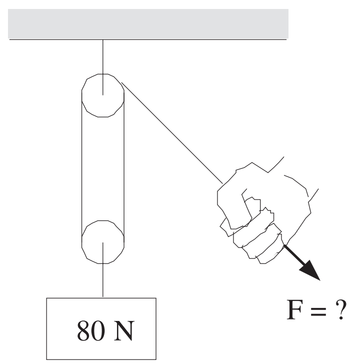
■ When a rope of neglibable mass passes through a massless, frictionless pulley, the tension of the rope will be the same on each side of the pulley. The angle of the rope as it enters or leaves the pulley does not effect the tension.

The following examples demonstrate this principle for stationary systems.

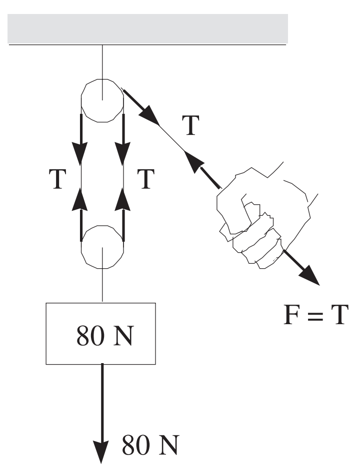


question ►

What force is required to hold the block below in a stationary position? Assume the rope to be of neglibable mass.



solution:



Since the 80 N block is at rest, the net force on it is zero:

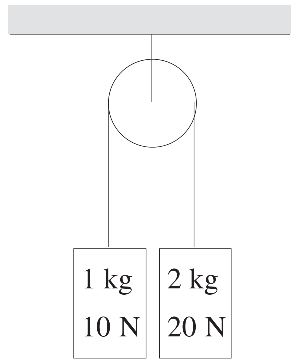
$\Sigma F_{80N} = 0$, $T + T - 80\ N = 0$ $T = 40\ N$.

The force applied by the hand must equal the tension, F = 40 N.◆

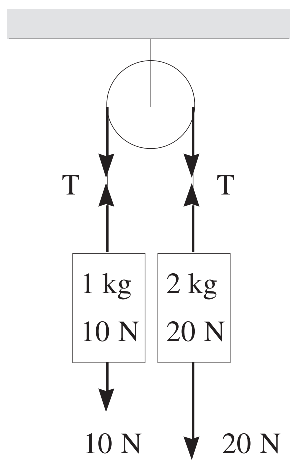
The following question deal with a system under acceleration.

question ►

What is the acceleration and tension in the system below? Assume the rope and pulley to be of negligable mass, and that the pulley rolls without friction. Let g = 10 m/s^2.



solution:



Since the pulley experiences greater downward force on its right side it will turn in a clockwise direction. This will give the 10 N block a positive (upward) acceleration and the 20 N block a negative acceleration:

$\Sigma F_{10N} = ma$, $\quad T - 10\ N = 1\ kg \cdot a$, $\quad T - 10 = 1a$.

$\Sigma F_{20N} = m(-a)$, $\quad T - 20\ N = -2\ kg \cdot a$, $\quad T - 20 = -2a$.

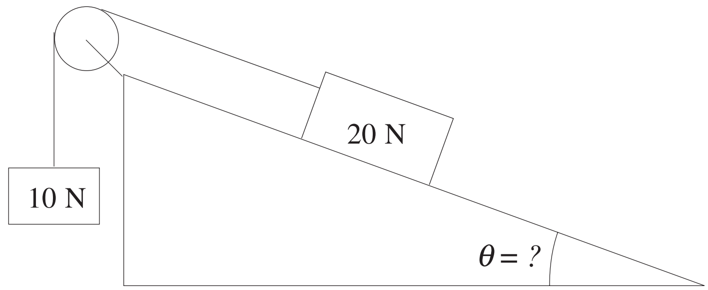
If we subtract the second equation from the first (please note the sign change in second equation) we get:

$$\begin{array}{rl} & T - 10 = 1a \\ + & -T + 20 = 2a \\ \hline & 10 = 3a \end{array} \qquad a = 3.3\ m/s^2$$

The value of *T* may be found by plugging the value for *a* back into either equation above. T = 13.3 N.◆

question ►

At what angle θ, will the system below, have a minimum value of f_s.



$\Sigma F_{10N} = 0$, $T - 10 = 0$, $T = 10$.

$\Sigma F_{20N} = 0$, $20 \sin \theta - T \pm f_s = 0$.

If we let $f_s = 0$, and substitute 10 in for T, we get:

$\sin \theta = 0.5$, $\theta = 30°$.◆

assignment ►

Referring back to the previous question, which of the following will NOT occurr as the angle θ is gradually increased beyond 30°?

A. The 10 N block will rise.

B. The normal force will decrease.

C. The force of friction will increase and then decrease.

D. The tension will increase and then decrease.

solution:

Recall that as θ increases the sin of θ deceases. Since the normal force is equal to mg·sin θ, FN decreases.

As FN decrease the tension and the force of static friction both increase to keep the 20 N block stationary.

Answer C , to be competed later.

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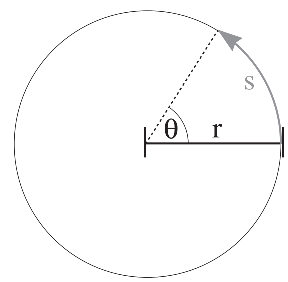
Uniform Circular Motion

■ **Uniform circular motion** occurs when an object moves in a circular path at constant speed.

3.1 Angular Quantities



Angular Displacement

Angular displacement is a measure of the angle θ though which a body in circular motion rotates. By definition a counterclockwise rotation is positive, and a clockwise rotation is negative. The SI unit of angular displacement is the radian (rad). A radian is defined as the arc length s divided by the radius r.

$$\boxed{\theta_{(rad)} = \frac{s}{r}}$$

Equation 3.1

Angular displacement may also be expressed in degrees. The following relationships between degrees and radians are explained in more detail in section #.

$$360° = \frac{2\pi r}{r} = 2\pi \text{ rad} \qquad 1 \text{ rad} = \frac{360°}{2\pi \text{ rad}} = 57.3°$$

Angular Velocity

Angular velocity ω is a measure of the rate of rotation. It is defined as the angular displacement over time, and has the SI units of rad/s, but may also appear as degrees/s, or rotations/min (rpm).

$$\boxed{\omega = \frac{\Delta\theta}{\Delta t}}$$

Equation 3.2

assignment ►

If a wheel rotates at 200 rpm for 30 s what is the angular displacement of the wheel given in $\pi \cdot$rad?

solution:

At first the unit of a $\pi\cdot$rad may have thrown you, but hopefully you stuck with it to get:

$$30\text{s} \times \frac{\text{min}}{60\text{ s}} \times \frac{200\text{ rev}}{\text{min}} \times \frac{2\pi\cdot\text{rad}}{\text{rev}} = 200\pi\cdot\text{rad} \blacklozenge$$

Angular Acceleration (NOT ON THE MCAT)

Angular acceleration α is the change in angular velocity over time.

►As far as MCAT preparation is concerned you may safely ignore all references and equations which relate to angular acceleration in your physics textbook.

Period and Frequency

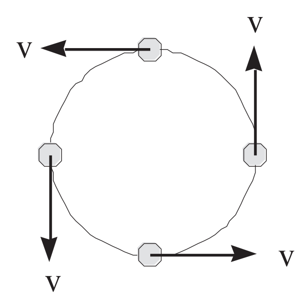
The time to complete one revolution is called the **period**, *T*.

$$\boxed{T = \frac{\text{time}}{\text{revolution}}}$$

The reciprical of the period is called the **frequency** *f.* Frequency is a form of angular velocity typically expressed in revolutions (a.k.a. cycles) per second.

$$\boxed{f = \frac{1}{T}} \qquad \boxed{f = \frac{\text{revolutions}}{\text{time}}}$$

Tangential Velocity and Speed

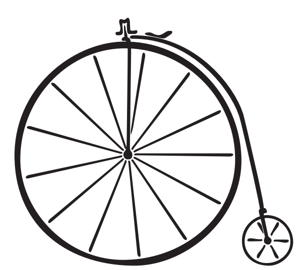


The velocity vector although constant in magnitude, changes direction as an object follows a circular path.at constant speed. In the adjaccent diagram the direction and magnitude of the instantaneous velocity is shown. Note that the direction of the velocity is always *tangent* to the circle, and that the magnitude of the velocity is equal to the speed. The instantaneuos velocity vector of an object in circular motion is referred to as **tangental velocity** v_T.

The speed, *v,* may found by dividing the circumference of the circle by the period:

$$\text{speed (v)} = \frac{\text{distance}}{\text{t}} = \frac{\;\frac{2\pi r}{\cancel{\text{rev}}}\;}{\frac{\text{t}}{\cancel{\text{rev}}}}$$

Tangentual Velocity and Angular Velocity



When an old fashion bicycle is taken out for a spin the tangential velocity (a.k.a. the rolling velocity) of its two tires will be the same but the angular velocities are different. If we think about it, it seems that the smaller tire must have a greater angular velocity to keep up with the larger. In fact the relationship between v_T and ω is:

$$\boxed{v_T = r\omega}$$

Equation 3.3

Note that v_T and ω are directly proportional and that ω and r are inversly proportional (See section # .)

assignment ►

If the larger wheel of an old fasioned bicyle has six times the diameter of the smaller wheel what will be the relative angular velocity of the smaller wheel as compared to the larger wheel when the bicycle is moving at 3.14 m/s?

solution:

Easy way: If we realize that ω and r are inversely proportional we are forced to conclude that the smaller wheel with 1/6 the radius of the larger must have 6 times the angular velocity.◆

Harder way:

$$v_{T\ (small)} = v_{T\ (big)}$$

$$r_{big}\omega_{big} = r_{small}\omega_{small} \qquad \frac{\omega_{small}}{\omega_{big}} = \frac{r_{big}}{r_{small}} = \frac{6}{1}$$ ◆

Things you don't need to worry about:

- **The velocity**, since both wheels will always have the same tangential velocity.
- **Coverting from diameters to radii**, since: $diameter_{big}/diameter_{small} = r_{big}/r_{small}$.

3.2 Centripetal Acceleration and Force

Centripetal Acceleration

For an object in uniform circular motion, the change in the velocity's direction with respect to time is called the centripetal acceleration, a_c.

■ The **centripetal acceleration,** a_c, is a vector quanity which is always directed toward the center of a circular path. The magnitude of the a_c is given by:

$$\boxed{a_c = \frac{v^2}{r}}$$

where v is the speed, and r is the radius of the circular path.

question ►

A jet, moving at constant speed, takes 10 s to complete one revolution around a horizontal circular path having a radius 100 m. Estimate the centripetal acceleration of the jet.

solution:

r = 100 m, T = 10 s, v = 2πr/T.

$$a_c = \frac{v^2}{r} = \frac{\left(2\pi r/T\right)^2}{r} = \frac{4\pi^2 r}{T^2}$$

Substituting in for r and T, and estimating that $\pi^2 = (3.14)^2 \approx 10$, gives:

$$a_c \approx \frac{4(\sim 10)100}{100} \approx 40 \text{ m/s}^2.$$

Note that since π^2 is slightly less than 10, the correct answer will be slightly less than 40 m/s².◆

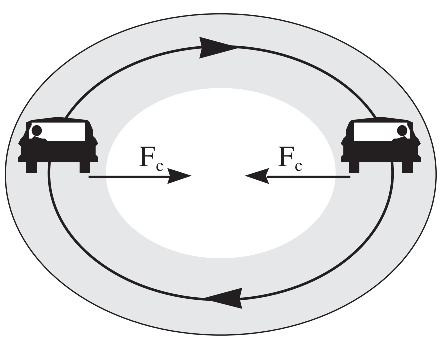
Centripetal Force

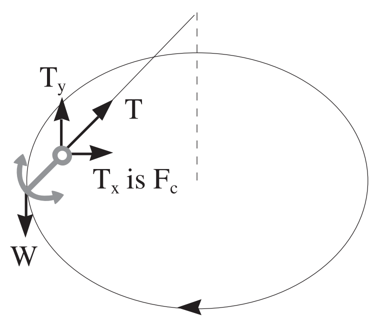
■ The **centripetal force**, F_c, is a vector quantity which is directed toward the center of a circular path. It is the centripetal force that causes the centtripetal acceleration, experienced during circular motion. The magnitude of F_c is given by:

$$\boxed{F_c = \frac{mv^2}{r}}$$

Any type of force that is directed toward the center of a circular path may act as the centripetal force.

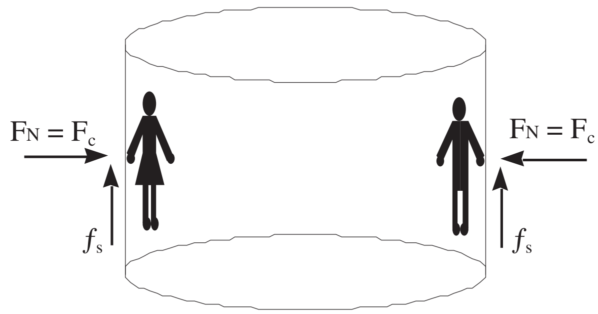
When a car travels on a level circular track, the component of static friction directed toward the center of the cirular motion is the centripital force. *The force between the tires and the road is static friction when the tires are rolling. Kinetic frictional forces are involved when the tires are locked and sliding occurs..*





When an anchor is tied to a rope and swung around in a horizontal circle, the component of the tension directed toward the center of the circular motion, T_x, is the centripital force. The vertical component of the tension, T_y, is equal and opposite to the anchor's weight.

An amusment park ride is shown below. People enter the inside of a broad cylinder which begins to rotate rapidly. When the speed of rotation reaches a certain value the floor of the cylinder is removed. The people are supported by the force of static friction, while the normal force causes the centripetal acceleration.



question ►

A child ties a 1 kg model plane on a 0.5 m string and swings it around in a *vertical* circle with an average speed of 4.0 m/s. What is the tension in the string when the plane is at its maximum height, mid-height, and minimum height? Let $g = 10\ m/s^2$. Assume minimal variations in speed.

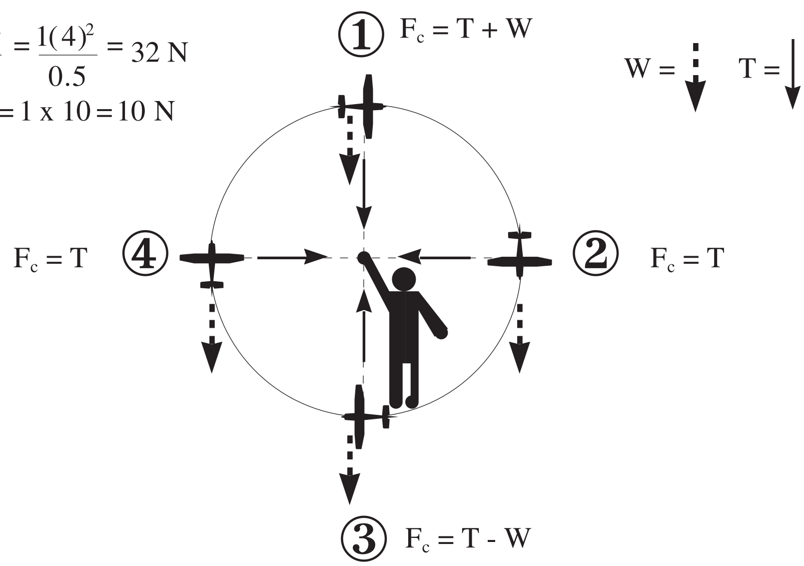
solution:

As shown below $F_c = 32$ N, and W = 10 N.

Forces directed toward the center of the circle are given positive values below.

$$F_c = \frac{mv^2}{r} = \frac{1(4)^2}{0.5} = 32\text{ N}$$

W = mg = 1 x 10 = 10 N



At point #1, the tension and the weight are both oriented toward the center of the circle. Each is a part of F_c. T = 32 N - 10 N = 22 N..

At points 2 and 4, the tension provides all of the centripital force, and so is equal to 32 N.

At point #3, the weight opposes the tension. The tension is greatest at this point because it must resist the force of gravity, and cause centripetal acceleration. T -= 32 N + 10 N = 42 N.◆

Rotational Equilibrium

■ An object which is in **rotational equilibrium** is stationary, or is engaged in uniform rotational motion.

4.1 Torque

Torque τ determines the magnetude and direction of rotaion resuling when a force is applied to an object. The SI unit of torque is $N \cdot m$. The sign of torque indicates direction. The tendency to rotate counterclockwise is assosiated with positive torques, while clockwise rotation is related to negative torques. In order for an object to remain in rotational equilibrium it must experience no net torque.

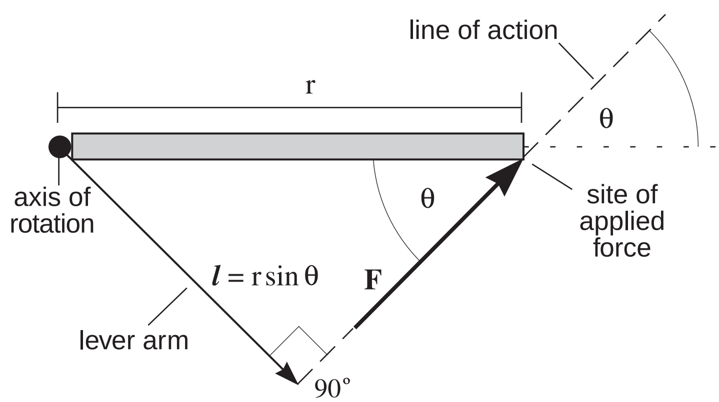


Figure 4.1

Figure #■ is an overhead view of a free swinging door. The *axis of rotation* corresponds to the door's hinges. When a force **F** is applied, the door will swing. The degree to which it does so depends on *r*, the distance between the axis of rotaion and the *site of applied force*, and θ, the angle made between r and F. The mathematical definition of torque is:

$$\tau = F \cdot r \sin\theta$$

Equation 4.1

Let's think about pushing that sliding door in Figure #■. From experience we expect the door will swing more readily if we push harder; push farther away from the hinges; and push perpendicular to the door. Now check out Equation #■. The torque is greatest when we maximize F and r and when θ approaches 90°.

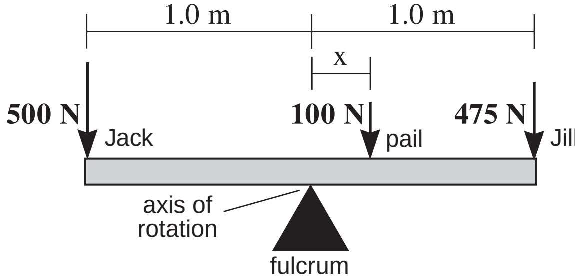
An alternative way to define torque involves the concept of a lever arm. The **lever arm** l is the perpendicular distance from the axis of rotation to the *line of action* (a line drawn through the force vector). Using l in place of r we get $\tau = F \cdot l$. Under this formulation, torque is maximized as F and l increase.

Before we see a numerical example of torque , you'll be happy to know that MCAT questions ussually have θ equal to 90°, so that torque will simply equal F•r.

example ►

Jack and Jill plan to sit on opposite ends of a 2.0 meter seesaw. Jack is 500 N and Jill is 475 N. How many cm from Jill should a 100 N pail of water be placed so that Jack and Jill are in rotational equilibrium?

solution:



Since all forces are perpendicular to r, θ = 90° and τ = F•r. Since Jack weighs more than Jill the position of the pail is placed on Jill's side of the seesaw.* The values for r are 1.0 m, 1.0 m, and x, for Jack, Jill, and the pail respectively. Setting the summation of torques equal to zero we get:

$$\tau_{Jack} - \tau_{Jill} + \tau_{pail} = 0.$$

The negative signs for Jill and the pail indicate torques which favor clockwise rotations. Plugging the numbers in gives:

$$(500)(1.0) - (475)(1.0) - x(100) = 0,$$

$$x = 25/100 = 1/4.$$

The pail must be placed 1/4 of a meter (25.0 cm) from the axis of rotation. This is a distance of 75.0 cm from Jill. ◆

**Had we placed the pail on Jack's side of the seesaw, we would have still arrived at the correct answer, but the sign of x would have been negative; either way it is inadvisable to seesaw next to a pail of water* |00–)] .

4.2 Center of Gravity (≈ center of mass*)

When an object is flung into the air, it rotates about a fixed point called its center of gravity. If we follow the object's trajectory (in the absence of air resistance) we'll find that though the object gyrates its center of gravity consistently follows the equations of motion (Section#■).

The **center of gravity** is a point from which the entire weight of an object can be imagined to originate. For uniform objects the center of gravity tends to be at the center. For objects made from different substances, or objects having complex shapes, the center of gravity will always lie closer to the greater concentration of weight. *The **center of mass** is a concept which is so closely related to the center of gravity, that the MCAT does not require that you tunderstand the difference.

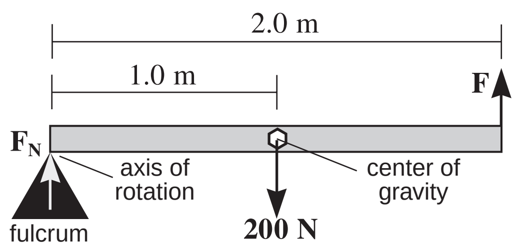
In the previous example we did not consider the weight of the seesaw because its center of gravity was at the axis of rotation. Since r, the distance from the axis of rotation to the center of gravity of the seesaw was zero, the torque was also zero.

■ No torque will result from any force applied at the axis of rotation.

example ►

A 2.0 m uniform beam of 200 N is in rotational and translational equilibrium. It is supported on its left edge by the normal force F_N , and on its right edge by the upward force F. What are the magnetudes of F_N and F?

solution:



Since the beam is uniform we may assign its center of gravity to the center of the beam. Since the system is in rotational equilibrium $\sum\tau = 0$. Setting the sumation of torques equal to zero and plugging in allows us to solve for F:

$$-(200)(1.0) + F(2.0) = 0, \qquad F = 100 \text{ N.} \blacklozenge$$

Since the system is in translational equilibrium $\sum F = 0$. Setting the sumation of forces equal to zero and plugging in allows us to solve for F_N:

$$F_N - 200 + 100 = 0, \qquad F_N = 100 \text{ N.} \blacklozenge$$

In a one dimensional system the center of gravity may be calculated by use of Equation#■ :

$$\text{center of gravity} = \frac{W_1 d_1 + W_2 d_2 + W_3 d_3 + \ldots}{W_1 + W_2 + W_3 + \ldots}$$

Equation 4.2

were *W* is the weight and *d* is the displacement of each component of a system. For example lets imagine that we have a meter stick that weighs 5.0 N. Now we glue a 3.0 N weight at the 40 cm mark and a 2.0 N weight at the 90 cm mark and we want to find the center of gravity of this system. If we let W_1 and d_1 represent the meter stick we get W_1 = 5.0 N and d_1 = 50 cm, remember the center of graity of the meter stick will be at its center. Also W_2 = 3.0 N and d_2 = 40 cm, W_3 = 2.0 N and d_3 = 90 cm. Plugging into equation#■ gives:

$$\text{center of gravity} = \frac{(5.0)(50) + (3.0)(40) + (2.0)(90)}{5.0 + 3.0 + 2.0} = 55 \text{ cm}.$$

Work and Energy

5.1 Work

■ **Work,** *W,* occurs when a force acting on an object causes the displacement of that object. Work is a scalar quantity defined as:

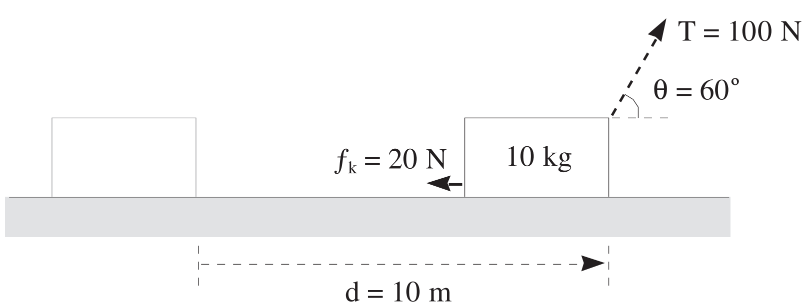
$$\boxed{W = Fd\cos\theta}$$

where *F* is the magnitude of the force, d is the displacement, and θ is the angle between the force and the displacement. The SI unit of work is the joule, *J*, and is equivalent to a N·m.

- Work can only occur when a displacement also occurs.
- Maximum work occurs when the force is parelell to the displacement ($\cos 0° = 1$). In this case the equation for work reduces to, W = Fd.
- A force acting perpendicular to the displacement does no work ($\cos 90° = 0$).
- Negative work is done when the force, or a component of the force, opposes the displacement vector ($\cos 180° = -1$).

example ►

In the diagram below the force of tension and kinetic friction are given.for an object undergoing a 10 m displacement. Calculate the the work done by the tension force, the work done by the frictional force, and the net work done on the object



solution:

The work done by the tension force is $W = (100\ N)(10\ m)(\cos 60°) = 500\ J$.

The work done by the frictional is $W_f = (20\ N)(10\ m)(\cos 180°) = -200\ J$.

The net work is obtained by adding the two values above:

$W_{NET} = W + W_f = 500\ N + (-200\ N) = 300\ J$◆

An alternative method of finding the net work in the preceding question is to multiply the net force which acts parallel to the displacement, by the displacement. In the example below T_x is the horizontal component of the tension.

$W_{NET} = (T_x + f_k)d$, $W_{NET} = (T \cos \theta + f_k)(d)$, $W_{NET} = (100 \cos 60° - 20)(10) = 300\ J$.

5.2 Energy

■ When positive net work is done on an object, that object gains in energy. **Energy** is the "currency" which is exchanged when one object does work on another. All forms of energy are scalar quantities. The SI unit of energy is the joule, *J*, and is equivalent to a N·m.

Kinetic Energy

The energy associated with moving objects is called **kinetic energy**, *KE*. Objects which move from one location to another have translational KE. Objects which are spinning have rotational KE. In this chapter only translational kinetic energy will be discussed

Translational KE is defined as:

$$KE = ½mv^2$$

where m is an object's mass, and v is its velocity or speed.

Work Energy Theorem

The **work energy theorem** states that the net work preformed on a object will be equal to the change in the kinetic energy of that object.

$$W_{NET} = \Delta KE$$

example ►

If the object in the previous question starts at rest, what is its velocity after it slides 10 m?

solution:

$W_{NET} = \Delta KE$, $W_{NET} = KE_f - KE_o$, $W_{NET} = ½mv^2 - ½mv_o^2$.

Since the object starts at rest, its initial velocity and its initial kinetic energy are equal to zero,

$$300\ N = ½(10\ kg)v^2 - 0.$$

Solving for the final velocity gives: $v = 7.7\ m/s$. ◆

Potential Energy

■ An object can have energy not only due to its motion, but also due to its shape or its position. This form of energy is called **potential energy**, *PE*, because it has the potential to do work.

When you stretch a rubber band, you are doing work on it. The rubber band will stores most of the energy that you put in. This energy is called elastic PE. When you release the rubber band suddenly, this energy is converted into KE.

When you lift a weight, you do work on it. The PE of the weight is directly proportional to its height. This energy is called gravitational PE. If you release the weight the gravitational force will do work on the weight, thus converting its PE into KE. In this chapter only gravitational PE will be discussed.

Gravitational PE is defined as:

$$\boxed{PE = mgh}$$

where mg is weight of an object, and h is its vertical height. In this equation the value of g is typically given a positive value.

assignment »

What is the change in potential energy when an 8 kg object is raised vertically by 5 m? Let $g = 10\ m/s^2$.

solution:

$$\Delta PE = mg\Delta h,$$

Since the height of the object is increasing, the value of h will be a positive number.

$$\Delta PE = (8\ kg)(10\ m/s^2)(5\ m) = 20\ J$$ _

Conservation of Mechanical Energy

In the absence of frictional forces it is found that the sum of the change in kinetic energy and potential energy is equal to zero,

$$\Delta KE + \Delta PE = 0.$$

Collectively these two forms of energy are called the total mechanical energy, ME. Mechanical energy is said to be conserved when the final mechanical energy is equal to the initial mechanical energy,

$$ME_f = ME_o.$$

assignment »

What will be the final velocity of an object that is dropped from rest from a height of 5 m? Let $g = 10\ m/s^2$. Assume frictional forces to be negligible.

solution:

$\Delta KE + \Delta PE = 0$, $\frac{1}{2}m(v^2 - v_o^2) + mg\Delta h = 0$.

Canceling out the terms for mass and substituting in for v_o, g, and Δh, gives:

$$\frac{1}{2}v^2 + (10\ m/s^2)(-5\ m) = 0.$$

Solving for the final velocity gives: v = 10 m/s. _

►Although the final velocity in the last question does not have a negative sign, the downward direction of the velocity vector can be inferred from the information given.

Nonconservation of Mechanical Energy

When frictional forces are significant, mechanical energy is not conserved. The ME lost to friction is equal to the work done by friction, W*f*. As we saw earlier, this value will be a negative quanity.

$$\Delta KE + \Delta PE = Wf$$

The work done by friction causes an increase in heat. Heat is a form energy associated with atomic vibration .

assignment »

When a 2 kg object suspended in a pool of water is released from rest, its velocity is measured to be 4 m/s, after it has dropped a vertical distance of 5 m. What is the work done by friction on this object? Let g = 10 m/s^2.

solution:

The work done by friction will be equal to the change in the total mechanical energy, $\Delta ME = Wf$.

$$\frac{1}{2}m(v^2 - v_o^2) + mg\Delta h = Wf$$

$$\frac{1}{2}(2\ kg)(4\ m/s)^2 + (2\ kg)(10\ m/s^2)(-5\ m) = Wf$$

$$16\ J - 100\ J = Wf,\quad Wf = -84\ J$$ _

►Since frictional force is always oriented 180° from the displacement vector, the value for the work done by friction should always be negative ($Wf = Ff \cdot d \cdot \cos 180°$). On the MCAT the negative sign assosiated with the work done by friction may not appear in the passage or in the answer choices. When this occurs, the negative sign is understood to be present.

Coservation of Total Energy

So far we have discused kinetic, potential, and heat energy. Future attractions will include: electromagnetic energy, electrical energy, and chemical energy. No matter what form energy takes the following two rules apply:

■ All forms of energy are interconvertable.

■ The total energy content always remains constant.

5.3 Power

■ Power, P, is a scalar quantity, equal to the amount of work preformed over time.

$$\boxed{P = W/t}$$

The SI unit of power is the watt, W, which is equal to a J/s.

An alternative definition of power is obtained by substituting Fd in for the term W to give, P = Fd/t. Since d/t is equal to velocity it follows that

$$\overline{P} = F\overline{v},$$

where $\overline{P}$ is the average power, F is a constant force, and $\overline{v}$ is the average velocity. (Since the definition of work used above is Fd rather than Fd cos θ, the velocity and the force must be parallel.)

question ►

A 1 kg block initial at rest on a frictionless horizontal surface is subject to a constant horizontal force of 2 N, what is the power output associated with this force?

solution:

The acceleration of the block is obtained from:

F = ma, 2 N = 1 kg·a, $a = 2\ m/s^2$.

d =	**v_o =**	**v =**	**a =**	**t =**
?	0 m/s	?	$2\ m/s^2$	Let t = 1 s

When solving for power the time interval, if not given, may be set equal to one second to simplify things. Using $d = v_o t + ½at^2$ gives x = 1 m.

- P = Fd/t = (2 N)(1 m)/1 s = 2 W.

Or we may use $v = v_o + at$, to get v = 2 m/s. Since the average velocity is equal to $½(v_o + v)$, we obtain an average velocity of 1 m/s.

- $\overline{P} = F\overline{v}$ = (2 N)(1 m/s) = 2 W ◆

? To be continued.