

## 2. *Electrostatics*

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**THIS IS A ROUGH DRAFT. Your comments are welcome: hochstim@netside.net**

**Electostatic force**, described below, refers to forces arising from stationary charges.

### **2.1 Charge and Coulomb's Law**

Charge is a funimental property of matter. Like energy, charge is a conserved quantity; in any physical or chemical process the sum total of all charges will remain constant

The charge on a proton is positive, while the charge on an electron is negative. The magnetude of the charge on the proton equals that of the electron. This quantity is known the **elementary charge**,  $e$ , (*a.k.a. funimental or unit charge*). The value of  $e$  in SI units is:

$$e = 1.60 \times 10^{-19} \text{ C}$$

where  $C$  is the SI unit of charge known as the **Coulomb**.

Like charges experience a repulsive electrostatic force when placed near each other, while unlike charges are subject to an attractive electrostatic force. The magnitude of this force may be found by applying **Coulomb's Law**:

$$F = k \frac{q_1 q_2}{r^2}$$

where  $k$  is a proportionality constant equal to  $9.0 \times 10^9 \text{ N}\cdot\text{m}/\text{C}^2$ ,  $q_1$  and  $q_2$  are the magnitude of the two charges in Coulombs,  $r$  is the separation between the charges in meters, and  $F$  is the **Coulombic force**, in Newtons.

*assignment* ►

### **2.2 Electric Fields**

#### **Electric Fields**

■ The **electric field**,  $E$ , is a vector quantity that describes the direction and magnetude of force that a charge will experience. The word *field* is use to describe quantities that are defined over an extended region of space rather than at a speciic point.

► **Electric fields are caused by the presents of charge**, so when a small positive charge known as a *test charge*,  $q_0$ , is placed near another charge,  $Q$ , an electric field emminating from  $Q$ , will cause an electrostatic force,  $F$ , on  $q_0$ , (and visa versa). The magnetude of  $E$  is given by:

$$\mathbf{E} = \frac{\mathbf{F}}{q_0}$$

where the SI units for the electric field are N/C, since  $F$  is in Newtons and  $q_0$  is in coulombs. **The direction of  $E$  is the same as the direction of  $F$ .** Remember that the test charge (which is used to *test* for the magnetude and direction of  $E$ ) is positive. Therefore, the direction of  $E$  will radiate away from a positive  $Q$ , and toward a negative  $Q$ . Shortly, we'll see this represented graphically.

The electric field strength at a distance  $r$  from a charge  $Q$ , may be found by use of equations#■ and#■:

$$F = k \frac{q_0 Q}{r^2}, \quad F = E q_0, \quad E q_0 = k \frac{q_0 Q}{r^2}, \dots$$

$$E = k \frac{Q}{r^2}$$

**Equation 3.1** Electric field due to a point charge  $Q$ .

The charge  $Q$  is called a point charge, since it is considered to occupy a single point in space. Note that  $E$  obeys the inverse square law, i.e.,  $E$  decrease by the inverse square of the distance from  $Q$ .

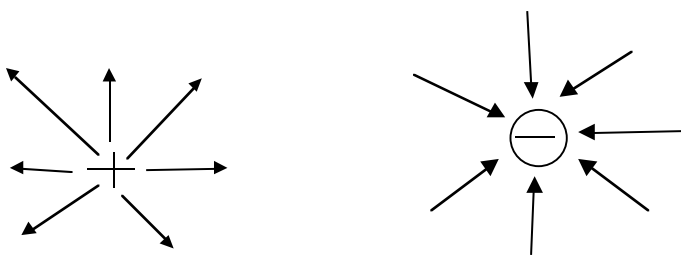
*assignment* ►

show vector nature

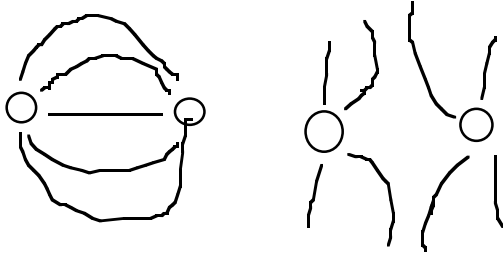
## Electric Field Lines

The direction and magnetude of an electric field may be represented by a series of arrows. These arrows are also known as lines of force, because they represent the direction and strength of the force caused by the electric field on  $q_0$ . A few simple rules concerning electric field lines follow:

- Field lines radiate away from positive charges and toward negative charges.
- As the field strenght increases, the seperation between field lines decreases.



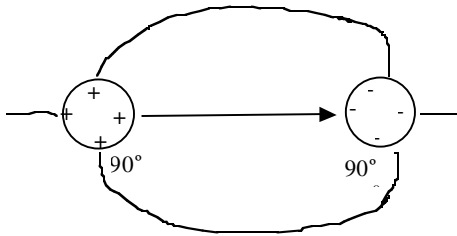
- When two or more charges are present, the electric field is the result of the vector sum of all individual field lines.



### Conductors and Insulators

**Conductors** are materials in which charge freely flows, while in **insulators** charge flows with difficulty, or not at all. **When a conductor takes on a charge, the excess charge distributes itself evenly throughout the surface of the object.** The reason for this behavior is based on Coulomb's law. Since like charge repel, the excess charges move as far away from each other as possible, i.e., to the surface. Since all the excess charge resides at the surface, the surface may be considered to be nothing more than a number of equally distributed individual point charges. From this follows the final rule regarding field lines:

► At the surface of a conductor, all field lines are perpendicular to the surface.



## 2.3 Electrical Potential Energy and Electrical Potential

### Electrical Potential Energy and Potential

We have seen that the gravitational potential energy of an object depends the mass of an object and its height. Analogously, the **electrical potential energy, *EPE***, depends on the charge on an object and its distance  $r$  from some location. Gravitational PE may be found by multiplying force (mg) times displacement (h). EPE is found the same way:

$$F = k \frac{q_1 q_2}{r^2}, \quad F \cdot r = k \frac{q_1 q_2}{r}$$

The electrostatic force (F) times the displacement (r) gives:

$$\boxed{EPE = k \frac{q_1 q_2}{r}}$$

Note that when  $q_1$  and  $q_2$  are like charges EPE is positive, and that when  $q_1$  and  $q_2$  are oppositely charged EPE is negative.

When a small positive test charge  $q_o$  is moved from infinity to a location near a positive charge  $Q$ , positive work  $W$  will need to be done against the repulsive electrostatic force. This work will be equal to the EPE which will also be positive. Conversely, when  $Q$  is negative,  $q_o$  will be attracted to  $Q$ . In this case  $W$  is still equal to the EPE, but both are negative because energy is released.

■ The **electrical potential**, or **potential**,  $V$ , is a scalar quantity defined as the EPE of  $q_o$  divided by the charge of  $q_o$ . Its value is given relative to infinity, at which point  $V$  is arbitrarily set to be zero.

$$\boxed{V = \frac{\text{EPE}}{q_o}} \quad \text{OR} \quad \boxed{V = \frac{W}{q_o}}$$

The SI unit of electrical potential is the volt (V). 1 volt is equal to 1 joule/coulomb.

► While EPE is a measure of energy, **potential is the energy per unit charge**.

The voltage due to  $Q$  can be found in a few easy steps (recall, work = force·displacement):

$$F = k \frac{q_o Q}{r^2}, \quad \text{EPE} = W = F \cdot r, \quad \frac{F \cdot r}{q_o} = \frac{kQ}{r}, \dots$$

$$\boxed{V = \frac{kQ}{r}}$$

**Equation 3.2** Voltage due to a point\* charge  $Q$   
 (\*A *point* charge refers to charge which is confined to a small volume.)

**Two important distinctions between the electric field and electric potential are:**

►  $E$  is a vector and  $V$  is a scalar

►  $E$  varies with the inverse square of distance, while  $V$  varies by the inverse of distance.

**assignment** ►

ask about where  $E$  and  $V$  is zero.

■ The **potential difference** or **voltage** is the change in the potential from one point  $A$ , to another point  $B$ . This may be expressed as  $V_{AB}$ , or simply as  $V$  (the same as potential!).

Imagine that  $q_o$  moves from point  $A$  to point  $B$ . The potential difference  $V_{AB} = \Delta V = V_B - V_A$ . Therefore:

$$\boxed{V_{AB} = \frac{\text{EPE}_{AB}}{q_o}} \quad \text{OR} \quad \boxed{V_{AB} = \frac{W_{AB}}{q_o}}$$

Where  $\text{EPE}_{AB} = \Delta \text{EPE} = \text{EPE}_B - \text{EPE}_A$  and  $W_{AB} = \Delta W = W_B - W_A$ , respectively. When  $q_o$  moves from one point to another, the quantity of work done (i.e.,  $\Delta \text{EPE}$ ) is independent of the path that  $q_o$  takes. From Equation#■, we can see that only the potential difference  $V_{AB}$  and the value of  $q_o$  are relevant in determining the work ( $W = \Delta V \cdot q_o$ ).

Finally we may use EQ#■ to derive an additional formulation of potential:

$$V_{AB} = kQ \left[ \frac{1}{r_B} - \frac{1}{r_A} \right]$$

In Equation#■ the potential difference between two points  $A$  and  $B$ , is given as a function of their distance  $r$ , from a point charge  $Q$ . **Note that when  $r$  remains constant there is no change in the potential around  $Q$ .**

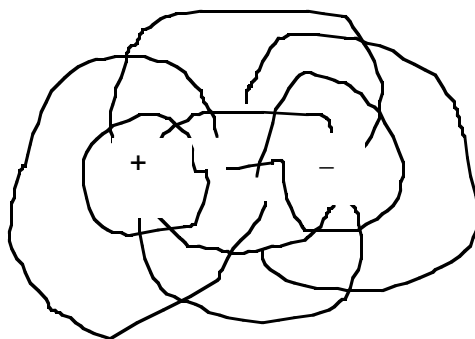
## Equipotential Surfaces and Lines

An **equipotential surface** is a surface with a constant electrical potential. An **equipotential line** is any cross-section cut through an equipotential surface. As an example imagine a point charge  $Q$  at the center of an infinite number of concentric spherical shells. According to Eq#■ each shell is an equipotential surface. Now imagine cutting a cross-section through these shells. Each circular ring is an equipotential line.

### Electric Fields Revisited

In Figure#■ equipotential lines and electric field lines are drawn for the cross-section of the space around a positive and a negative charge.

p 537 cut/John



You should note that the **equipotential lines are always perpendicular to the electric field lines**. This allows us to view the electric field in a new way. Instead of regarding them in terms of force per unit charge, we may instead view them as running from high to low potential. In fact instead of measuring the electric field strength in terms of  $N/C$ , we may find the average magnitude of the electric field by measuring the change in voltage per meter or:

$$\bar{E} = \frac{\Delta V}{\Delta r}$$

EQ#■ may be used to find the *average* electric field strength,  $\bar{E}$ , for regions with a varying  $E$ -field (as  $\Delta r$  is reduced  $\bar{E}$  approaches  $E$ ). For regions with a constant  $E$  an exact value may be found. The SI units for  $E$  as expressed in EQ#■ is  $V/m$ , which is equivalent to  $N/C$ .

### assignment ►

Do a diagram, at which point is  $E$ -field constant (where lines parallel)

## 2.4 Capacitors

### Conductors and Insulators

**Electrical conductors** are substances (usually metals) that readily allow the flow of electrons. **Insulators** are materials (usually nonmetals) which do not exhibit any appreciable degree of electron mobility.

### Capacitors

Capacitors are devices that store charge; they are composed of both conductors and insulators. The simplest type of capacitor is the parallel plate capacitor. It consists of two flat conducting plates which are separated by an insulating material known as a **dielectric**. The separation between the two plates  $d$  is constant. Each plate stores an equal but opposite quantity of charge  $q$ .

In Figure#■ we initially see an uncharged capacitor. Next we connect each plate to a battery, which removes electrons from one plate and deposits an equal number of electrons on the other. This causes the plates to become charged, resulting in a potential difference  $V$  which is equal to the voltage of the battery. **Capacitance**,  $C$ , is a measure of the amount of charge that can be stored as a function of  $V$ ,

$$C = \frac{q}{V}$$

where  $q$  is the quantity of charge in Coulombs on **one** plate. The SI unit of capacitance is the **farad**,  $F$ .  $1F = 1 \text{ Coulomb/Volt}$ . Because the farad is a very large unit, capacitance is often reported in microfarads, nanofarads, or picofarads ( $1 \mu\text{F} = 10^{-6} \text{ F}$ ;  $1 \text{ nF} = 10^{-9} \text{ F}$ ;  $1 \text{ pF} = 10^{-12} \text{ F}$ ).

#### *assignment* ►

If a 12 V battery is attached to a parallel plate capacitor with a plate area of overlap of  $0.05 \text{ m}^2$  and a separation of  $0.06 \text{ cm}^2$ . What will be the electric field strength at in the interior of the capacitor?  $E = V/d$  or  $\Delta V/\Delta r$

The value of the capacitance is a function of the area  $A$  over which the plates overlap, the separation between the plates  $d$ , and the electrical properties of the dielectric, which is represented by  $\kappa$  the dielectric constant.

$$C = \frac{\kappa \epsilon_0 A}{d}$$

The term  $\epsilon_0$  is a constant called the **permittivity of free space**, which is equal to  $8.85 \times 10^{-12} \text{ F/m}$ . The dielectric constant is a unitless number equal to one when the dielectric is a vacuum or air, and is greater than one for other substances.

#### *example* ►

show  $V$  and  $E$  for parallel plat cap. show how lines bend at end se fig 16-2 p378 Giancoli

## Dielectrics and the Electric field

### Energy Stored in a Capacitor

The quantity of energy stored in a capacitor may be found when the capacitance and voltage are known by the use of EQ#■.

$$\text{Energy} = \frac{1}{2}CV^2$$

## 2.5 Electrical Dipoles

Need to do see Shat p. 127 Also see John/Child p.457

## *Electric Circuits*

### 3.1 Direct Current

#### Current and Electromotive Force

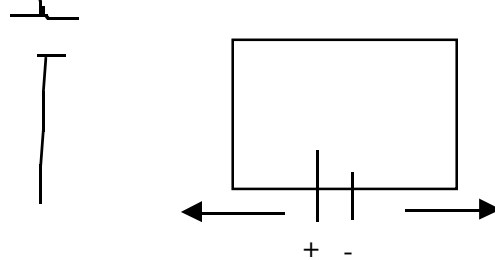
A **battery** is a device that stores electrical energy. Most batteries have two **terminals** (*conducting surfaces*), one positive (*high potential*), the other negative (*low potential*). When a conducting loop of wire is placed on each of a battery's terminals a circuit is formed. A **circuit** refers to any closed conducting loop. Due to a potential difference (voltage) arising from chemical reactions within the battery, electrons flow out of the negative terminal, through the wire, and into the positive terminal. This unidirectional flow of charge is called **direct current, I**. Mathematically, current is defined as the quantity of charge  $q$  that passes through a cross-section of a conductor in a given unit of time:

$$I = \frac{q}{t}$$

The SI unit of current is the **ampere A**.  $1 \text{ A} = 1\text{C/s}$ .

Before it was known that electrons were the charge carriers responsible for current, a hypothetical construction called **conventional current (a.k.a. current)** was adopted, and remains in use today. **Current** is imagined to be composed of *positive* charge carriers. As a result, **current flows in the opposite direction as electrons do**, i.e., current flows from high to low potential. Fig#■ depicts the direction of electron flow and of current.

p 554-555 cut/john also talk about how to depict the battery and about EMF



The voltage across the terminals of a battery when no current is present is called the **electromotive force** or **emf**,  $\mathcal{E}$ . In Fig#■ the battery has an emf of 12 volts.

## Resistance and Ohm's Law

Under ordinary circumstances the amount of current in a circuit is directly proportional to the voltage, or  $V \propto I$ . For a given voltage the magnitude of the current depends on the *resistance*. The **resistance**,  $R$ , is analogous to friction; the greater the resistance, the more electrical energy is converted into heat. As a result the current is greatest when the voltage is large and the resistance is low, or  $I = V/R$ . This expression is known as **Ohm's law** and is commonly written as:

$$V = IR$$

where the SI units of  $R$  are given in ohms,  $\Omega$ .  $1 \Omega = 1 \text{ volt/ampere (V/A)}$ . Components in a circuit that have a significant resistance are called **resistors**.

### Internal Resistance

As current moves through a circuit the energy per unit charge (that's the voltage) falls due to resistance. This resistance is present not only in the wire, but also in the battery. When a battery generates a current, the voltage across its terminals (**terminal voltage**) is less than its emf. This is due to resistance within the battery called **internal resistance**,  $R_{IN}$ . Lets use Ohm's law to solve for  $R_{IN}$ :

### example ►

A 12.0 V battery is found to have a terminal voltage of 11.8 V and a current of 0.50 A when a wire forms a circuit at the battery's terminals. What is the battery's internal resistance?

### solution:

To solve for  $R_{IN}$  we may rearrange Ohm's Law:

$$R_{IN} = \frac{\Delta V}{I}$$

$\Delta V$  is the drop in voltage due to  $R_{IN}$ , or  $12.0 \text{ V} - 11.8 \text{ V} = 0.20 \text{ V}$ ;  $I$  is given as 0.50 A, so:

$$R_{IN} = \frac{0.20 \text{ V}}{0.50 \text{ A}} = 0.40 \Omega \blacklozenge$$



**assignment ►**

What is the resistance of the wire in the previous example?

**solution:**

The voltage drop across the wire is 11.8 V. The current is 0.50 A. Using Ohm's law to solve for R we get:

$$R = \frac{11.8 \text{ V}}{0.50 \text{ A}} = 23.6 \Omega \blacklozenge$$

**►Referring to example and assignment above:**

- Since the current had only one path to follow, the same quantity of charge must flow through both the battery and in the wire. Current is analogous to the flow rate of water. If there is only one pipe, the quantity of water flowing through the pipe must be identical through out the pipe.
- Resistance in a circuit acts like a series of waterfalls. The drop in voltage ( $\Delta E_{PE}/q$ ) is analogous to the drop in pressure ( $\Delta PE_{grav}/\text{volume}$ ) as water loses potential energy.
- The battery acts like a pump increasing the potential (pressure) of the current. When the current has completed its path around the circuit the drop in voltage is equal to the emf of the battery or:

$$V_{\text{total}} = \Delta V_{\text{IN}} + \Delta V_{\text{wire}}$$

$$V_{\text{total}} = IR_{\text{IN}} + IR_{\text{wire}}$$

$$V_{\text{total}} = I(\Delta V_{\text{IN}} + \Delta V_{\text{wire}})$$

$$12 = 0.5(0.4 + 23.6)$$

- In this example the resistance of the battery and the wire were of central importance, however **in most cases the resistance of the battery and the wire are negligible**. On the MCAT you may ignore these terms unless you are given specific information about them, or reason to believe that these variables are relevant to the question.

**Variables effecting Resistance**

When water moves through a long and narrow pipe there is a greater resistance to flow than when a shorter pipe with a larger diameter is used. By analogy, long wires with small diameters offer greater resistance to the flow of current than shorter wires with a larger cross-sectional area do. This relationship is quantified below:

$$R = \rho \frac{L}{A}$$

Where  $L$  and  $A$  are the length and cross-sectional area of the conducting material, respectively. The **resistivity**,  $\rho$ , is a proportionality constant dependent on the conducting material and its temperature. Note that while resistivity is a constant for a given material at a specific temperature, resistance depends on the resistivity and the geometry of the conducting substance.

**Power**

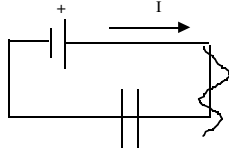
When current flows through a resistor power is dissipated in the form of heat. The power loss may be found by:

$$P = IV = I^2R = \frac{V^2}{R}$$

where  $V$  is the voltage drop across the resistor (*not necessarily across the whole circuit*).

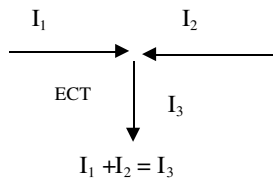
## Circuit Diagrams

The following symbols are conventionally used to represent components within a circuit:

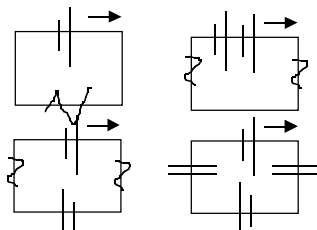


## 3.2 Kirchoff's Rules

► **Junction Rule:** At any junction in a circuit the quantity of current flowing in will equal the quantity of current flowing out. This situation depicted in Fig#■ is analogous two water pipes converging; the volume flow rate in pipe 3 will be equal to the volume flow rate in pipe 1 plus pipe 2.

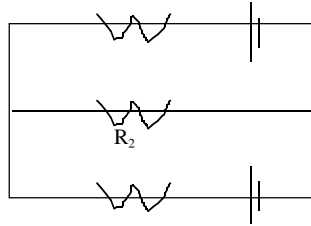


► **Loop Rule:** The rise in potential equals the drop in potential around any closed circuit loop. By analogy, we may imagine water being pumped up hill. The water pressure increases as a result of increased gravitational potential energy. As the water flows around and back down to the pump it returns to its original position and pressure. As you may have guessed the pump is analogous to the battery, and the pressure is analogous to the potential. Some examples are given below.



*example* ►

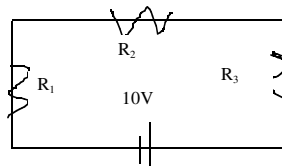
What is the voltage drop across resistor  $R_2$ ?



### 3.3 Series and Parellel Wiring

#### Resisters in Series

When circiut components are connected in a single file arrangement they are said to be wired **in series**. The three resisters below are wired in series.



■ *Since each resister lies on the same path without any branching, the current that runs through each resister must be the same. As current passes through the battery its potential is raised by ten volts. As the current passes through each resister its potential drops, in accord with Kirchokov's loop rule.*

#### *assignment ►*

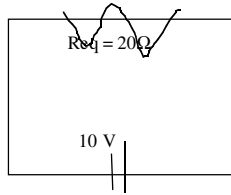
Referring to Fig #■, if the voltage drop across  $R_1$  is 2V, and across  $R_2$  is 3V, what is the voltage drop across  $R_3$ ?

*solution: need to solve it loop rule*

**Resistances in series are additive.** This means that we may reduce the three resistors in Fig#■ to one *equivalent resistor*. An **equivalent resistor** is an imaginary circuit componet that is used to simplify a circuit diagram. The imaginary resistance through an equivalent resistor is called the **equivalent resistance**,  $R_{eq}$ . In this case we may say  $R_{eq} = R_1 + R_2 + R_3$ , or for the more general case:

$$R_{eq} = R_1 + R_2 + R_3 \dots$$

Referring again to Fig#■, if  $R_1$ ,  $R_2$ , and  $R_3$  are 4.0  $\Omega$ , 6.0  $\Omega$ , and 10  $\Omega$  respectively, we may now redraw the circuit to include just one equivalent resisitor in place of  $R_1$ ,  $R_2$ , and  $R_3$ :

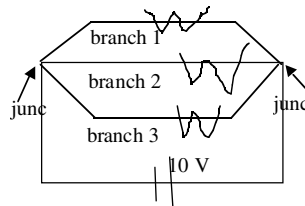


If we wished to solve for the total power consumed by this circuit we could simply use the battery's emf and the  $R_{eq}$ , to give:

$$P = \frac{V^2}{R_{eq}}, \quad P = \frac{(10 \text{ V})^2}{20 \Omega} = 5 \text{ W}$$

## Resisters in Parrelel

“Branches” exist where ever a circuit junction is present. In order for components to be wired **in parrelel** only one component may be present on a branch, and each branch must begin and end at the same junctions. The three resisters below are wired in parrelel.



■ *Since each resister lies on parelel paths the voltage drop across each resister must be the same.* In this case the voltage drop from junction 1 to junction 2 will be 10 V, therefore all resistors will share the same 10 V drop

*assignment* ► *need solve it junction rule rule*

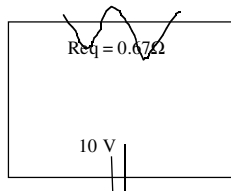
Referring to Fig#■, if the battery geneates a curent of 20 A, and 8 A pass through  $R_1$  and 4 A pass through  $R_2$ , what is the current through  $R_3$ ?

**Resistances in parelel are added as recipricals.** This means that we may reduce the three resistors in Fig#■ to one equivalent resistor by use of the following equation:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \dots$$

Refering again to Fig#■, if  $R_1$ ,  $R_2$ , and  $R_3$  are each  $2.0 \Omega$ , we may now redraw the circuit to include just one equivalent resistor in place of  $R_1$ ,  $R_2$ , and  $R_3$ :

$$\frac{1}{R_{eq}} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}, \quad \frac{1}{R_{eq}} = \frac{3}{2}, \quad R_{eq} = \frac{2}{3} \text{ or } 0.67 \Omega$$



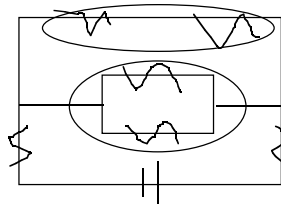
Note that had these three resistors been wired in series the equivalent resistance would have been larger ( $1.5 \Omega$  rather than  $0.67 \Omega$ ). This result can be generalized: **When a group of resistors are wired in series, their equivalent resistance will always be greater than when they are connected in parallel.**

## Resistors in Complex Circuits

In circuits containing resistors connected in both series and parallel, the two methods illustrated above are both used to solve for the equivalent resistance. Here's an example:

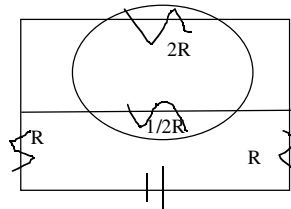
*example* ►

In the circuit below all resistances are equal to  $0.50 \Omega$ , and the emf of the battery is  $12.0 \text{ V}$ . Solve for the current, and for the power dissipated.

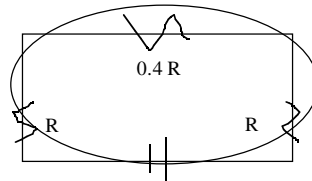


*solution:*

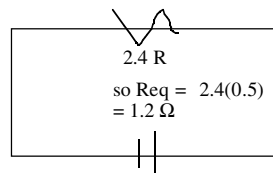
First we ect. (since all R's are same lets leave them as R for now)



Then ect



then



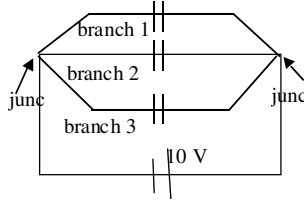
The current may now be found to  $I = V/R = 12/1.2 = 10 \text{ A}$ . &  $P = IV = (10)(12) = 120 \text{ W}$ . or  $I^2R$  or  $V^2/R$ .

**assignment** ►

Ask about solving for voltage drop , current, and power for one o more resistors.

**Capacitors in Parrelel**

The three capacitors below are wired in parrelel:



■ Since each capacitor lies on parelel paths the voltage drop across each capacitor must be the same. In this case the voltage drop from junction 1 to junction 2 will be 10 V, therefore all capacitors will share the same 10 V drop

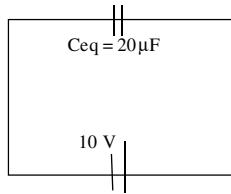
**assignment** ►

If the one of the three capacitors in Fig #■ has a capacitance of  $1.0 \times 10^{-6}$  F what is the magnitude of the charge stored on each plate of this capacitor?

**Capacitances in parrelel are additive.** This means that we may reduce the three capacitors in Fig#■ to one *equivalent* capacitor. An **equivalent capacitor** is an imaginary circuit componet that is used to simplify a circuit diagram. The imaginary capacitance through an equivalent capacitor is called the **equivalent capacitance**,  $C_{eq}$ . In this case we may say  $C_{eq} = C_1 + C_2 + C_3$ , or for the more general case:

$$C_{eq} = C_1 + C_2 + C_3 \dots$$

Refering again to Fig#■, if  $C_1$ ,  $C_2$ , and  $C_3$  are  $4.0 \mu\text{F}$ ,  $6.0 \mu\text{F}$  and  $10 \mu\text{F}$  respectively, we may now redraw the circuit to include just one equivalent resisor in place of  $C_1$ ,  $C_2$ , and  $C_3$ :

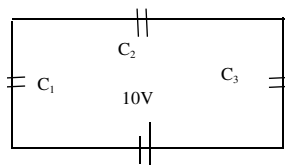


If we wished to solve for the total charge stored in this circuit we could simply use the battery's emf and the  $C_{eq}$ , to give:

$$q = C_{eq} \cdot V, \quad q = (20 \times 10^{-6} \mu\text{F})(10 \text{ V}) = 2.0 \times 10^{-4} \text{ C}$$

**Capacitors in Series**

The three capacitors below are wired in series.



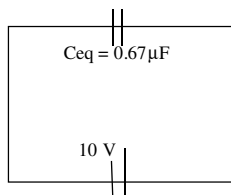
■ Since each capacitor lies on the same circuit, the quantity of charge transferred from the battery to each plate of each capacitor must be the same. The current that is produced by the battery causes equal quantities of charge to be deposited and removed from each plate of each capacitor. As the capacitors are charged to capacity the flow of current slows and eventually stops due to increasing Coulombic forces. Note that while  $q$  will be the same for all capacitors, the voltage drop across each capacitor will be inversely proportional to the capacitance ( $q = C \cdot V$ ).

**Capacitances in series are added as reciprocals.** This means we may reduce the three capacitors in Fig#■ to one equivalent capacitor though the use of the following equation:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \dots$$

Referring again to Fig#■, if  $C_1$ ,  $C_2$ , and  $C_3$  are each  $2.0 \mu\text{F}$ , we may now redraw the circuit to include just one equivalent capacitor in place of  $C_1$ ,  $C_2$ , and  $C_3$ :

$$\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}, \quad \frac{1}{C_{eq}} = \frac{3}{2}, \quad C_{eq} = \frac{2}{3} \text{ or } 0.67 \mu\text{F}$$

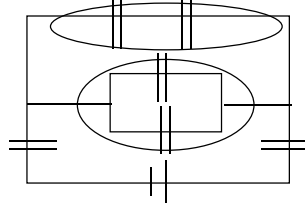


Note that had these three capacitors been wired in parallel the equivalent capacitance would have been larger ( $1.5 \mu\text{F}$  rather than  $0.67 \mu\text{F}$ ). This result can be generalized: **When a group of Capacitors are wired in parallel, their equivalent capacitance will always be greater than when they are connected in parallel.**

## Capacitors in Complex Circuits

### assignment ►

In the circuit below all capacitances are equal to  $0.50 \mu\text{F}$ , and the emf of the battery is  $12.0 \text{ V}$ . Solve for the the total charge and power stored by this system.



## Summary

Component:	Wired in:	All share the same:	Formula:
Resistors	Series	Current	$R_{eq} = R_1 + R_2 + R_3 \dots$
Resistors	Parrelel	Voltage	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \dots$
Capacitors	Parrelel	Volatage	$C_{eq} = C_1 + C_2 + C_3 \dots$
Capacitors	Series	Charge	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \dots$



## 3.4 An Aqueous Analogy

### INSTRUCTIONS:

The following section may help solidify your understand of Chapters# &■; if not, skip it.

A common analogy is the comparison of the flow of water to the flow of charge (curent). Here's a list of aqueous terms followed by their electrical analogs ( the “≡” symbol means *is equivalent to*):

*The volume of water flowing through a pipe in a given unit of time is analogous to the quantity of charge flowing through a wire in a given unit of time.*

**1. volume of H<sub>2</sub>O** (vol)  
≡ **quantity of charge** (q)

**2. flow rate** ( $\Delta\text{vol}/\Delta\text{time}$ )  
≡ **curent** ( $\Delta q/\Delta\text{time}$ )

*The gravitational PE of water, ( $GmM/r$ ), or ( $mgh$ ) for small values of  $\Delta h$ , is analogous to the electric PE ( $kqQ/r$ ), where  $G$  and  $k$  are constants,  $m$  and  $M$  are masses, and  $q$  and  $Q$  are charges.*

**3. gravitational potential energy** ( $\text{PE}_{\text{grav}}$ )  
≡ **electrical potential energy** (EPE)

*Absolute pressure is analogous to potential, while the change in pressure is analogous to voltage.*

**4. absolute pressure** ( $\text{PE}_{\text{grav}}/\text{vol}$ )  $\rightarrow (\rho gh = mgh/\text{vol} = \text{PE}_{\text{grav}}/\text{vol})^*$   
≡ **potential** (EPE/q)

**5.  $\Delta$ pressure** ( $\Delta\text{PE}_{\text{grav}}/\text{vol}$ )  $\rightarrow (\rho g\Delta h = mg\Delta h/\text{vol} = \Delta\text{PE}_{\text{grav}}/\text{vol})$   
≡ **voltage** ( $\Delta\text{EPE}/q$ )

*The following analogs (dams, waterfalls, resistance, and pumps) complete the list:*

**6. dams** store vol  
≡ **capacacitors** store q

**7. waterfalls** decrease  $\text{PE}_{\text{grav}}$  causing a decrease in pressure.  
≡ **resisters** decrease EPE causing a decrease in voltage

**8. resistance** increases in a pipe of smaller diameter  
≡ **resistance** increases in a wire of smaller diameter

**9. pumps** (increase pressure)  
≡ **batteries** (increase potential)

\* “ $\rho$ ” refers to density in this context.

## 3.5 Alternating Current

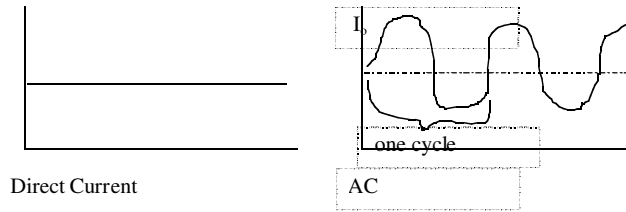
### AC Current

Unlike direct current, which flows in one direction, **alternating current** oscilates back and forth. The most common form of alternating current is sinusoidal i.e., is described by the *sine* function. Specifically:

$$I = I_o \sin(2\pi ft)$$

where  $I_o$  is the maximum value of the current,  $I$  is the value of the current at time  $t$ , and  $f$  is the frequency of oscilation. Recall that since  $2\pi f = \omega$ , Eq# may be condensed to:

$$I = I_o \sin(\omega t)$$



Unlike direct current which always has a positive value of  $I$ , alternating current is bidirectional and thus has both positive, negative and zero values for current at different times. Since  $P = I^2 R$ , the power delivered by an alternating current will be greatest for  $I_o$  and zero when  $I = \text{zero}$ . Note that since  $I$  is squared, the power, although continuously changing, will never be negative.

## RMS Current and Voltage

Since  $V = IR$ , voltage is directly proportional to current. Therefore  $V = V_o \sin(2\pi ft)$ .

Since  $P = IV$ ,  $P = I_o V_o \sin(2\pi ft)$ .

What's the point? Well, since  $I$ , and  $V$ , and  $P$ , are all constantly changing, we need a simple mathamatic method for dealing with this. This method involves using **root mean square current and voltage**,  $I_{rms}$ ,  $V_{rms}$ , to average out these changes. Root mean square current and voltage are defined as:

$$I_{rms} = \frac{I_o}{\sqrt{2}}, \quad V_{rms} = \frac{V_o}{\sqrt{2}}$$

When working with alternating currents we may use the same equations as we did with direct currents as long as we use  $I_{rms}$  for the current and  $V_{rms}$  for the voltage. Ohm's law becomes:

$$V_{rms} = I_{rms} R$$

and the **average power**,  $\bar{P}$ , is:

$$\bar{P} = I_{rms} V_{rms} = I_{rms}^2 R_{rms} = \frac{V_{rms}^2}{R_{rms}}$$

Finally,

$$\bar{P} = I_{rms} V_{rms} = \left( \frac{I_o}{\sqrt{2}} \right) \left( \frac{V_o}{\sqrt{2}} \right) \quad \text{So...}$$

$$\bar{P} = \frac{1}{2} I_o V_o$$

## *Magnetic Fields*

**Magnetic fields**,  $B$ , are similar to electric fields, both are vector quantities, that describe the magnetude and direction of force that a charge will experience. Magnetic field lines are used to visualize the dirction and strength of the magnetic field in a similar manner as is done with electric fields. Unlike the electric field, the magnetic field can only exert a force on a *moving* charge.

Charged particals can produce both electric and magnetic fields. While a stationary charge generates only an electric field,  $E$ , a moving charge produces both an electric and a magnetic field. Magnetic fields are also found surrounding certain metalic (typically iron) objects that have been previously exposed to powerful magnetic fields. Such objects are known as **perminant magnets**.

## 4.1 Magnetic Force

### Force on a Moving Charge

When a charged particle  $q$ , moves at a velocity  $v$ , through a uniform magnetic field  $B$ , a **magnetic force**  $F$  is exseted on the particle, given by,

$$F = qvB \sin \theta$$

where the magnetic field strenght  $B$  in SI units is the **tesla**,  $T$ .  $1T = 1 \text{ N/A}\cdot\text{m}$ , and  $\theta$  refers to the angle between the particle's velocity vector and the direction of the magnetic field. Fig #■ depicts a method (Right Hand Rule-1, *RHR-1*) for determining the direction of  $F$ .

explain  $\theta$  and  
.....  
.....out of paper  
and  
xxxx into paper  
xxxxx

Using your right hand if you point your thumb in the direction of the velocity vector of a positively charged particle, and your fingers in the direction of  $B$ , your palm will face in the direction of  $F$ . For a negatively charged particle the direction of  $F$  will be reversed. Note that the angle  $\theta$  is represented by the angle made between your thumb ( $v$ ) and fingers ( $B$ ). If  $v$  is parrelel to  $B$  no force is exerted, but when  $v$  and  $B$  are perpendicular  $F$  is at a maximum.

If a positively charged particle moving to the North encounters a uniform magnetic field directed upward the particle will experience a force  $F$  directed to the west, as depicted in Fig#■. Since  $v$  and  $B$  ae pependicular  $\sin \theta$  will be equal to one, and  $F = qvB$ . In response to  $F$  the partical will accelerate by continually changing the direction, but not the magnetude, of its velocity vector. As it does the direction of  $F$  will continuously change as well. The end result is the particle exhibits uniform circular motion.

PIC HERE  
SHOW NORTH

see p602 Cut/John

Since  $F$  is always directed toward the center of motion we may say that the magnetic force also acts as the centripetal force, or:

$$\mathbf{F}_{\text{mag}} = \mathbf{F}_c$$

$$qv\mathbf{B} = \frac{mv^2}{r}$$

$$\boxed{r = \frac{mv}{q\mathbf{B}}}$$

where  $r$  is the radius of the circular path.

### assignment ►

A particle of mass  $m$  moving horizontally encounters a uniform magnetic field of  $B$  directed downward. The particle begins to rotate clockwise in a circle of radius  $r$ . What is the charge (including the sign) on the particle?

*solution:*

## Force on a Current-Carrying Wire

Eq#1 states that a charge which moves through a magnetic field will experience a force, unless  $\theta$  is  $0^\circ$  or  $180^\circ$ . Therefore, it follows that a current (moving charges) in a wire will result in a net force on the wire as long as the wire does not run parallel to the magnetic field. The magnitude of this force is given by:

$$\boxed{F = ILB \sin \theta}$$

where  $I$  is the current,  $L$  is the length of the wire, and  $\theta$  is the angle made between  $B$  and  $I$ . Fig #2 demonstrates the use of RHR-1 for Eq#1. Recall that the direction of  $I$  is always given for the conventional current, which is composed of imaginary positive charge carriers.

Insert Fig#2

## 4.2 Magnetic Fields Produced by Current

### Long Straight Wires

A current  $I$  in a long straight current carrying wire will produce a circular magnetic field whose strength is given by:

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

where  $\mu_0$  is a constant called the **permeability of free space** =  $4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}$ ,  $I$  is the current, and  $r$  is the perpendicular distance from the wire. The direction of  $B$  is determined through the use of Right Hand Rule-2, RHR-2, below.

Insert Fig#3 RHR-2

## Loops of Wire

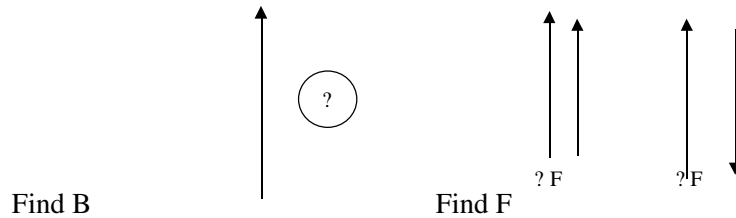
A current  $I$  in a loop of wire will also produce a circular magnetic field whose strength at the center of the loop is given by:

$$\mathbf{B} = \frac{\mu_0 IN}{2r}$$

where  $N$  is the number of coils (turns) of wire, and  $r$  is the radius of the loop.

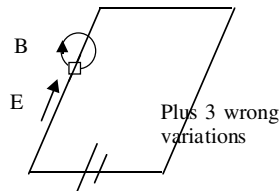
Insert Fig## RHR-2

*example* ►



*example* ►

Which of the following correctly depicts the direction of the electric and magnetic fields?



## 4.3 Current Produced by Magnetic Fields

Current moving through a conductor generates a magnetic field; conversely the movement of a conductor through a magnetic field can generate current. When a conducting material cuts through the lines of a magnetic field, the charged particles in that conductor experience a magnetic force. When a wire cuts across a uniform magnetic field at constant velocity, a direct current of constant magnitude is generated. When a loop of wire is rotated at constant angular velocity across a magnetic field, a sinusoidal alternating current results.

Possibly a passage on a rail gun and another on a generator could be included here.